ANALYSIS AND SYNTHESIS OF ERRORS IN MECHANISMS

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CERTIFICATE

This is to certify that the work "Analysis and Synthesis of Errors in Mechanisms" by S.S. Gavane has been carried out under my supervision and has not been submitted elsewhere for a degree.

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ABSTRACT

The analysis and control of errors in precision mechanisms is extremely important in order to obtain the desired level of accuracy. In this work three types of mechanisms, namely, Geneva mechanisms, cam follower system and gear train are considered for mechanical error analysis and synthesis. Both deterministic and probabilistic procedures are presented for predicting the effect of various tolerances and clearances on the acceleration and jerk of external and internal Geneva mechanisms. The effect of changes in geometrical and other system parameters on the kinematic and dynamic response of a cam follower system is found stochastically. The probability characteristics of the integrated gear train position error of gear train are determined using the method presented by Michalec. The problem of allocation of tolerances and clearances (or errors) in mechanisms is formulated as a constrained optimisation problem and nonlinear programming techniques are used for the solution. This procedure distributes the tolerances so as to minimize a measure of the manufacturing cost while satisfying the specified requirements on the deviation of the output parameter of the mechanism. The present study is expected to offer an unified method of error analysis and synthesis for all mechanical systems.

CHAPTER 1

INTRODUCTION

1.1 IMPORTANCE OF ERROR ANALYSIS

The development of science and technology is closely connected with the creation of modern high precision mechanisms, machines and other sensitive devices which produce the parameters of motion with high accuracy. Mechanisms are applied widely in measuring devices, computing devices, recording and registration equipment, optical instruments, automatic-controlling devices and similar others.

It is well known that in a real mechanism, the actual dimensions will be different from the theoretical ones. The difference will be caused by the limitations of the manufacturing processes, poor workmanship, inaccuracies of mechanism assembly and the clearances in the bearings of kinematic pairs which provide relative displacements of links. If the tolerances and clearances exceed a particular limit, then the very purpose of the mechanism will be lost. For example, if the tolerances and clearances exceed a particular limit in a Geneva mechanism, then the roller may enter the slot with a large value of jerk, which will ultimately result in poor performance of the mechanism. In the case of internal combustion engines with valves, the lift of the valves, which is governed by a cam-follower

system, should be known very accurately so as to clearly define the closing and opening of the valves. Otherwise it may result in the inefficient use of the fuel. Similarly, in the case of gear-train used in precision mechanisms, the various mechanical errors have to be controlled in a proper manner in order to obtain the desired level of precision in terms of motion, power transmission and dynamic response.

1.2 LITERATURE SURVEY

Several attempts have been made to predict and control mechanical errors in mechanisms in the past. Hartenberg and Denavit |1| presented a deterministic method for analysing the error in four-bar planar mechanisms. This method does not take into account the effect of clearances. A deterministic method has been presented by Kolhatkar and Yajnik |2| for estimating the effect of clearances on the output of a mechanism. Tuttle |3| used **vectorial** addition to analyze errors in cam operated devices and four-bar linkages. This method gave the approximation only of the error in a non-continuous system, and is not applicable to error analysis of cam-follower mechanisms. Rothbart |4| used a finite difference method to obtain the effect of surface roughness on follower acceleration (deterministically). The effect of manufacturing errors on the integrated gear train position error of a gear train was considered deterministically by Michalec |5|.

In this work the total error was computed by considering the contributions of individual errors as additive. Hence this method gives a conservative estimate of the total error.

A statistical method for the mechanical error analysis of planar four-bar mechanisms was offered by Garett and Hall |6| . This method yielded some useful results and paved the way for further research in that direction. This method was computer oriented and a lack of rigorous mathematical treatment makes it unsuitable for use in synthesis. Dhande and Chakraborty |7| have provided a stochastic model of the four bar linkage considering tolerances and clearances. The mechanical error was analyzed for the three-sigma band of confidence level. The problem of allocation of tolerances and clearances for a four-bar mechanism in an optimal manner was also considered by them. Rao and Ambekar |8| presented the error analysis of spherical function generating mechanisms by using a probabilistic approach. Dhande and Chakraborty employed a stochastic model by considering the kinematics of spatial cam mechanism, to analyse the three-sigma band of error in the follower displacement showing maximum and minimum errors |9| . Kim and Newcombe |10| extended the method of reference |9| to enable an analysis of the error in velocity and acceleration due to any given manufacturing

tolerances on the cam-profile. A probabilistic error analysis of gear trains was presented by Michalec |5| . By idealizing the gear train as a weakest link kinematic chain, Rao presented a method of designing gear trains with respect to bending strength and surface wear resistance |11| .

A model, which has been named as an impact pair, has been made by Dubowsky and Freudenstein |12| to analyze, deterministically, the dynamic response of mechanical system with clearances, by considering the displacement-forced motion, which is quasi-harmonic. While discussing this paper (at the end of Ref. |12|) Erdman and Sandor commented that at high speeds, the varying inertia forces in the mechanism as well as the strains due to elasticity of links, will certainly affect the forces on the impact pair. In other words, there will not only be displacement-forced motion, but other forces, which will be functions of parameters such as length and stiffness of links, configuration, distribution of mass and input speed of the mechanism, will also be there. The idea of considering the effect of errors on the dynamic response of cam-follower systems, presented in Chapter 3 of this thesis, germinated from these comments.

1.3 SCOPE OF THE PRESENT WORK

In this work, the analysis and synthesis of mechanical error is considered for the following mechanisms:

(1) External and internal Geneva mechanisms

(2) Cam-follower systems

(3) Gear-trains

In the case of Geneva mechanisms, firstly, the effects of tolerances on acceleration and jerk have been studied deterministically. After that, taking into account the clearances as well, a stochastic approach has been used and the three-sigma band widths of acceleration and jerk are plotted. Finally, the tolerances and clearances are allocated optimally with upper bounds on the standard deviations of acceleration and jerk.

In the investigation of cam-follower system, a probabilistic approach has been employed to analyze the effect of manufacturing errors in cam profiles and other geometrical parameters on the kinematic response. The variations in other system parameters such as equivalent mass, stiffness and damping are also considered to find the probabilistic characteristics of the dynamic response of the cam-follower system. The synthesis problems, involving the optimal allocation of errors, have also been solved by placing upper bounds on the standard deviations of the responses.

In the case of gear-train problem, the probabilistic error analysis presented in Ref. |5| has been used while solving the synthesis problem.

1.4 METHODOLOGY

1.4.1 For Deterministic Error Analysis:

For any mechanism, the equation relating the system parameters (a_1, a_2, ..., a_n), input variable (θ) and the output variable (ϕ) can be expressed in a general form as

$$f\,(a_1,\ a_2,\ \ldots,\ a_n;\ \theta\ ;\ \phi) = 0 \qquad (1.1)$$

If Δa_i ($i = 1, 2, \ldots, a_n$) represents the error in the system parameter a_i, the error in the output variable ($\Delta\phi$) for any fixed value of θ can be expressed as

$$\bar{f} = f\,(a_1 + \Delta a_1,\ a_2 + \Delta a_2,\ \ldots\ a_n + \Delta a_n;\ \theta;\ \phi + \Delta\phi) = 0 \qquad (1.2)$$

If $\{\Delta a_i\}$ represent the tolerances or clearances, then $\Delta\phi$ is called the mechanical error. Equation (1.2) can be expressed using Taylor's series expansion as

$$\bar{f} = f + df + \ldots = f\,(a_1,\ a_2,\ \ldots,\ a_n;\ \theta;\ \phi) + \sum_{i=1}^{n} \left.\frac{\partial f}{\partial a_i}\right|_{(a_1, a_2, \ldots; \theta)} \cdot \Delta a_i + \left.\frac{\partial f}{\partial \phi}\right|_{(a_1, a_2, \ldots; \theta)} \cdot \Delta\phi + \ldots = 0 \qquad (1.3)$$

For small values of Δa_i, $df = 0$ and hence the mechanical error ($\Delta\phi$) in the mechanism can be found as

$$\Delta\phi = -\left(\sum_{i=1}^{n} \frac{\partial f}{\partial a_i}\Big|_m \cdot \Delta a_i\right) / \left(\frac{\partial f}{\partial \phi}\Big|_m\right) \qquad (1.4)$$

where the notation $|_m$ has been used to denote the evaluation of the quantity at $(a_1, a_2, \ldots, a_n; \theta)$.

1.4.2 For Probabilistic Error Analysis:

The various tolerances and clearances in the mechanism have been assumed to be random variables. In the 7
absence of any information, these random variables can be taken to be normally distributed. As the output variable (ϕ) of the mechanism (like displacement, velocity and acceleration) is a non-linear function of the system parameters (including those for which tolerances and clearances are specified), it can be expressed as

$$\phi = \phi\,(a_1, a_2, \ldots, a_n; \theta) \qquad (1.5)$$

Thus ϕ will also be a random variable. As the theory of random variables is well established for linear functions, the Taylor's series expansion about the mean values of the random parameters is used to linearize the function ϕ. Since the sum of several random variables 11
following normal distribution is also a normal **variate,**
ϕ can also be treated as a normally distributed variable,

whose mean ($\mu_\emptyset$) and standard deviation ($\sigma_\emptyset$) are given by

$$\mu_\emptyset = \emptyset(\mu_{a_1}, \mu_{a_2}, \ldots, \mu_{a_n}; \theta) \tag{1.6}$$

and
$$\sigma_\emptyset^2 = \left\{ \sum_{i=1}^{n} \left(\frac{\partial \emptyset}{\partial a_i} \right) \Big|_{(\mu_{a_1}, \mu_{a_2}, \ldots, \mu_{a_n}; \theta)} \right\}^2 \cdot \sigma_{a_i}^2 \tag{1.7}$$

where μ_{a_i} and σ_{a_i} denote the mean and the standard deviation of a_i, respectively. Although the output variable $\emptyset$ has been assumed to be normally distributed on the basis of linearization, it can also be justified from central limit theorem |13| .

1.4.3 For Synthesis Problem:

The synthesis problem, involving the allocation of tolerances and clearances in the mechanisms, has been solved as a nonlinear optimization problem. As the manufacturing costs increase very rapidly as the tolerances and clearances to be maintained, approach zero, the objective function for minimization can be taken as the sum of the reciprocals of the tolerances and clearances. This function will represent, though not the exact manufacturing costs, at least a measure of the manufacturing costs.

By taking the tolerances and clearances as the design variables x_i, upper and lower bounds are placed on each x_i depending on the manufacturing limitations.

Further, bounds are also placed on the deviation of the output variable. Thus the aim of the optimization problem becomes one of minimization of the cost, that is, maximization of the tolerances and clearances, keeping the deviations of the output variables well within the specified limits. It is important to note that the results of optimization can not be applied directly for practical designs. The tolerances and clearances given by the optimization method have to be rounded off to nearest feasible discrete values.

For the solution of the constrained optimization problem, the interior penalty function has been chosen, coupled with variable metric method of unconstrained minimization and the cubic interpolation method of one dimensional minimization |14| . If the optimization problem is stated as,

$$\begin{aligned} &\text{Find } \vec{X}, \text{ which minimizes } f(\vec{X}) \\ &\text{and satisfies } g_j(\vec{X}) \leq 0, \quad j = 1, 2, 3, \ldots, m \end{aligned} \tag{1.8}$$

the following iterative process can be used in the interior penalty function method.

(i) Start with an initial feasible point $\vec{X}_1$ satisfying all the constraints with strict inequality sign, that is, $g_j(X_1) < 0$ for $j = 1, 2, \ldots, m$, and an initial value of $r_1 > 0$. Set $k = 1$.

(ii) Minimize $\phi(\vec{X}, r_k) = f(\vec{X}) - r_k \sum_{j=1}^{m} \frac{1}{g_j(\bar{X})}$,

by using the variable metric method and obtain the solution $\bar{X}_k^*$.

(iii) Test whether $\bar{X}_k^*$ is the optimum solution of the original problem. The convergence criterion is as follows:

(a) The process is assumed to have converged whenever the relative difference between the values of the objective function obtained at the end of any two consecutive unconstrained minimizations falls below 0.00001, or

(b) The process is to be terminated when the difference between the optimum points $\bar{X}_k^*$ and $\bar{X}_{k-1}^*$ becomes very small as given below

$$|(\Delta\vec{X})_i| \leq 0.00001 \qquad (1.9)$$

where $\Delta\bar{X} = \bar{X}_k^* - \bar{X}_{k-1}^*$ and $(\Delta X)_i$ is the ith component of the vector $\Delta\vec{X}$.

(iv) Find the value of the next penalty parameter, r_{k+1}, as

$$r_{k+1} = 0.1\,(r_k) \qquad (1.10)$$

(v) Set the new value of $k = k + 1$, take the new starting point as $\bar{X}_1 = \bar{X}_k^*$ and go to step (ii).

1.5 ORGANISATION OF THE THESIS

The analysis and synthesis of external and internal Geneva mechanisms is given in Chapter 2. Chapter 3 deals with the analysis and synthesis of the kinematic as well as the dynamic response of the cam-follower system. The synthesis of the gear-train is investigated in Chapter 4. Conclusions and recommendations have been included as Chapter 5. Appendix A contains the partial derivatives of acceleration and jerk of a Geneva mechanism, needed for the synthesis problem of Chapter 2. The probabilistic analysis of gear train errors constitutes appendix B. References are given at the end of the thesis.

CHAPTER 2

GENEVA MECHANISMS

2.1 GENERAL DESCRIPTION:

Geneva mechanisms have long been popular as a means for producing positive incremental motion because of their simplicity, both in design and construction, which makes them relatively low cost indexing devices. In a Geneva mechanism, the input shaft rotates at a constant speed while the output shaft turns through a suitable angle and then dwells for a given interval before repeating an identical motion. The output shaft always rotates in the same direction.

(i) External Geneva Mechanism:

Figure 2.1.a shows an external Geneva mechanism where

D = driving or input wheel,

F = follower or output wheel,

L = locking arm, and

2ψ = indexing period of the driver.

The centre line of the slot should be tangential to the pitch circle of the pin at the points of engagement and disengagement. The minimum number of slots is three and in a practical mechanism it is advisable to make the number of slots higher than the absolutely minimum number.

More number of slots give less noise |17|. Unfortunately, the locking of the wheel during the dwell period becomes less secure with larger number of slots. The mean acceleration of the follower (that is when no tolerances and clearances are considered) is governed by the number of slots. The indexing period of the Geneva mechanism shown in Fig. 2.1.a with four slots is equal to 90^o.

(ii) Internal Geneva Mechanism:

In the internal Geneva mechanism shown in Fig. 2.1.b, D = driving wheel, P = pin or roller on the driving member, F = follower or output wheel, L = locking arm, and 2ψ = indexing period of the driver. The indexing period of this internal Geneva mechanism is 270^o.

External Geneva drives have short indexing periods and hence the accelerations will be large because the acceleration is inversely proportional to the square of the indexing period. An internal Geneva drive has a long indexing period and hence the magnitude of the dynamic force (which is proportional to acceleration) is considerably lower than in the case of an external Geneva drive having the same number of slots. The character of the acceleration curve for an internal Geneva mechanism limits its application to low speeds only. The values of jerk are much smaller in the case of internal Geneval mechanism than in the case of external Geneva mechanism. The motion

of an internal Geneva mechanism near the central position can be considered to be uniform for all practical purposes as shown in Fig. 2.1.c |18| .

2.2 DETERMINISTIC ERROR ANALYSIS OF EXTERNAL GENEVA MECHANISM:

The following nomenclature is common to both external and internal Geneva mechanisms:

a_o = centre distance between the driving wheel and the follower with no tolerances and clearances,

b_o = distance between the centre of the pin and the driving wheel with no tolerances and clearances,

a = nominal centre distance = a_o,

b = nominal distance between the centre of the pin and the driving wheel = b_o,

c = width of the slot

$\Delta a, \Delta b$ = tolerances in the link lengths a and b respectively,

Δc = clearance in the slot width

$b + \Delta b$ = b_1 = actual length between the pin and the driving wheel,

$a + \Delta a$ = a_1 = actual length between the pin and the follower

α = input angle

β = output angle

Δc = c_1

$$\beta_1 = \beta + \Delta\beta$$

$$A = \frac{d^2\beta_1}{d\alpha^2}$$

$$ACC = \frac{d^2\beta_1}{dt^2} = \text{angular acceleration}$$

$$= \left(\frac{2\pi N}{60} \right)^2 . A$$

$$J = \frac{d^3\beta_1}{d\alpha^3}$$

$$JERK = \frac{d^3\beta_1}{dt^3}$$

$$= \left(\frac{2\pi N}{60} \right)^3 . J$$

N = speed of the driver (r.p.m.).

Figure 2.2.a shows, schematically, the roller in an intermediate position in the slot. Here it is assumed that the roller always remains in contact with the slot and that it never floats in it. From the geometry of the mechanism, the following relation can be derived 2 .

$$\sin\beta_o = \frac{b_o \sin\alpha}{(a_o^2 + b_o^2 - 2\, a_o\, b_o \cos\alpha_o)^{1/2}} \tag{2.1}$$

Figure 2.2.b shows the mechanism with tolerances and clearance. The following geometrical relations can be derived

$$\sin(\alpha + \beta + \Delta\beta) = \Delta C/x \tag{2.2}$$

$$\frac{x + b + \Delta b}{\sin(\beta + \Delta\beta)} = \frac{a + \Delta a}{\sin\left[\pi - (\alpha + \beta + \Delta\beta)\right]} \tag{2.3}$$

Substituting the value of x from Eq. (2.2) into Eq. (2.3) and solving, it can be shown that

$$\sin\beta_1 = \frac{b_1 \sin\alpha}{(a_1^2 + b_1^2 - 2\, a_1\, b_1 \cos\alpha)^{1/2}} + \frac{c_1\,(a_1 - b_1 \cos\alpha)}{(a_1^2 + b_1^2 - 2\, a_1\, b_1 \cos\alpha)} \tag{2.4}$$

where

$$a_1 = a + \Delta a$$
$$b_1 = b + \Delta b$$
$$c_1 = \Delta c$$

The second and the third derivatives of the output angle with respect to the input angle can be derived as

$$\frac{d^2\beta_1}{d\alpha^2} = \frac{A_1'' + B_1''}{\left[1 - (A_1 + B_1)^2\right]^{1/2}} + \frac{(A_1 + B_1)\,(A_1' - B_1')^2}{\left[1 - (A_1 + B_1)^2\right]^{3/2}} \tag{2.5}$$

where

$$A_1 = b_1 \sin\alpha / T \tag{2.6}$$
$$A_1' = b_1 \cos\alpha / T - a_1\, b_1^2 \sin^2\alpha / T^3 \tag{2.7}$$
$$B_1 = c_1\,(a_1 - b_1 \cos\alpha) / T^2 \tag{2.8}$$
$$B_1' = b_1\, c_1 \sin\alpha / T^2 - 2\, a_1\, b_1\, c_1 \sin\alpha \,.\, (a_1 - b_1 \cos\alpha) / T^4 \tag{2.9}$$

$$A_1'' = - b_1 \operatorname{Sin}\alpha / T - 3 a_1 b_1^2 \operatorname{Sin}\alpha \operatorname{Cos}\alpha / T^3 + 3 a_1^2 b_1^3 \operatorname{Sin}^3\alpha / T^5 \qquad (2.10)$$

$$B_1'' = b_1 c_1 \operatorname{Cos}\alpha / T^2 - \left[4 a_1 b_1^2 c_1 \operatorname{Sin}^2\alpha + 2 a_1 b_1 c_1 \operatorname{Cos}\alpha (a_1 - b_1 \operatorname{Cos}\alpha) \right] / T^4 + 8 a_1^2 b_1^2 c_1 \operatorname{Sin}^2\alpha (a_1 - b_1 \operatorname{Cos}\alpha) / T^6 \qquad (2.11)$$

$$T = (a_1^2 + b_1^2 - 2 a_1 b_1 \operatorname{Cos}\alpha)^{1/2} \qquad (2.12)$$

$$\frac{d^3\beta_1}{d\alpha^3} = (A_1''' + B_1''') / Y + 3 (A_1 + B_1) (A_1' + B_1')(A_1'' + B_1'')/Y^3 + (A_1' + B_1')^3 / Y^3 + 3 (A_1 + B_1)^2 (A_1' + B_1')^3 / Y^5 \qquad (2.13)$$

Here

$$Y = \left[1 - (A_1 + B_1)^2\right]^{1/2} \qquad (2.14)$$

$$A_1''' = b_1 \operatorname{Cos}\alpha / T + (4 a_1 b_1^2 \operatorname{Sin}^2\alpha - 3 a_1 b_1^2 \operatorname{Cos}^2\alpha)/T^3 + 18 a_1^2 b_1^3 \operatorname{Sin}^2\alpha \operatorname{Cos}\alpha / T^5 - 15 a_1^3 b_1^4 \operatorname{Sin}^4 / T^7 \qquad (2.15)$$

$$B_1''' = - b_1 c_1 \operatorname{Sin}\alpha / T^2 + 2 c_1 a_1^2 b_1 \operatorname{Sin}\alpha / T^4 - 14 a_1 b_1^2 c_1 \overset{\operatorname{Cos}\alpha}{\wedge} \sin\alpha / T^4 + 24 a_1^2 b_1^2 c_1 \sin\alpha \cdot \operatorname{Cos}\alpha (a_1 - b_1 \operatorname{Cos}\alpha) / T^6 + 24 a_1^2 b_1^3 c_1 \operatorname{Sin}^3\alpha / T^6 - 48 a_1^3 b_1^3 \operatorname{Sin}^3\alpha \cdot c_1 (a_1 - b_1 \operatorname{Cos}\alpha) / T^8 \qquad (2.16)$$

The individual effect of tolerances and clearance on the acceleration and the jerk is shown in Figs. 2.3 to 2.8 for an external Geneva mechanism having 4 slots with a crank length of 10 cm. From Figs. 2.3 to 2.5, it can be observed that the effect of Δc on the acceleration is much less than the effects of Δa and Δb. Hence a greater variation can be tolerated in the slot width compared to the variations in the crank length and the centre distance. It is to be noted that in these figures, the points lying on the Y - axis are singular because the Y - coordinate (which represents the percentage change in acceleration) blows up for zero mean acceleration (denominator) at $\alpha = 0$. Figures 2.6 to 2.8 show that the variation of jerk does not follow a definite pattern. Here also the tolerances in the crank length and the centre distance can be seen to be more important. The maximum percentage changes in acceleration and jerk for various values of Δa, Δb and Δc are given in Table 2.1.

2.3 DETERMINISTIC ERROR ANALYSIS OF INTERNAL GENEVA MECHANISM:

Figures 2.2.c and 2.2.d show an internal Geneva mechanism without and with tolerances. By proceeding as in the case of external Geneva mechanism, it can be derived that

$$\mathrm{Sin}\,\beta_o = \frac{b_o \ \mathrm{Sin}\,\alpha}{(a_o^2 + b_o^2 + 2\ a_o\ b_o\ \mathrm{Cos}\,\alpha)^{1/2}} \tag{2.17}$$

$$\mathrm{Sin}\,\beta_1 = \frac{b_1\ \mathrm{Sin}\,\alpha}{(a_1^2 + b_1^2 + 2\ a_1\ b_1\ \mathrm{Cos}\,\alpha)^{1/2}} + \frac{c_1\ (a_1 + b_1\ \mathrm{Cos}\,\alpha)}{(a_1^2 + b_1^2 + 2\ a_1\ b_1\ \mathrm{Cos}\,\alpha)} \tag{2.18}$$

$$\frac{d^2\beta_1}{d\alpha^2} = \frac{A_1'' + B_1''}{\left[1 - (A_1 + B_1)^2\right]^{1/2}} + \frac{(A_1 + B_1)(A_1' - B_1')^2}{\left[1 - (A_1 + B_1)^2\right]^{3/2}} \tag{2.19}$$

where

$$A_1 = b_1\ \mathrm{Sin}\,\alpha / T \tag{2.20}$$

$$B_1 = c_1\ (a_1 + b_1\ \mathrm{Cos}\,\alpha) / T^2 \tag{2.21}$$

$$A_1' = b_1\ \mathrm{Cos}\,\alpha / T + a_1\ b_1^2\ \mathrm{Sin}^2\alpha / T^3 \tag{2.22}$$

$$A_1'' = -b_1\ \mathrm{Sin}\,\alpha / T + 3\ a_1\ b_1^2\ \mathrm{Sin}\,\alpha\ \mathrm{Cos}\,\alpha / T^3 + 3\ a_1^2\ b_1^3\ \mathrm{Sin}^3\alpha / T^5 \tag{2.23}$$

$$B_1 = c_1\ (a_1 + b_1\ \cos\alpha) / T^2 \tag{2.24}$$

$$B_1' = -c_1\ b_1\ \mathrm{Sin}\,\alpha / T^2 + 2\ a_1\ b_1\ c_1\ \mathrm{Sin}\,\alpha \,.\, (a_1 + b_1\ \mathrm{Cos}\,\alpha) / T^4 \tag{2.25}$$

$$B_1'' = -\ b_1\ c_1\ \mathrm{Cos}\,\alpha / T^2 - 4\ a_1\ b_1^2\ c_1\ \mathrm{Sin}^2\alpha / T^4 + 8\ a_1^2\ b_1^2\ c_1\ \mathrm{Sin}^2\alpha (a_1 + b_1\ \mathrm{Cos}\,\alpha) / T^6 + 2\ a_1\ b_1\ c_1\ \mathrm{Cos}\,\alpha\ (a_1 + b_1\ \mathrm{Cos}\,\alpha)/T^4 \tag{2.26}$$

$$T = (a_1^2 + b_1^2 + 2\ a_1\ b_1\ \mathrm{Cos}\,\alpha)^{1/2} \tag{2.27}$$

$$\frac{d^3\beta_1}{dc^3} = (A_1''' + B_1''') / Y + 3\,(A_1 + B_1)\,(A_1' + B_1')(A_1'' + B_1'')/Y^3 + (A_1' + B_1')^3 / Y^3 + 3\,(A_1 + B_1)^2\,(A_1' + B_1')^3 / Y^5 \qquad (2.28)$$

where

$$Y = \left[1 - (A_1 + B_1)^2\right]^{1/2} \qquad (2.29)$$

$$A_1''' = - b_1 \cos\alpha / T + (3\,a_1\,b_1^2 \cos^2\alpha - 4\,a_1\,b_1^2 \sin^2\alpha) / T^3 + 18\,a_1^2\,b_1^3 \sin^2\alpha \cos\alpha / T^5 + 15\,a_1^3\,b_1^4 \sin^4\alpha / T^7 \qquad (2.30)$$

$$B_1''' = b_1\,c_1 \sin\alpha / T - 2\,a_1^2\,c_1\,b_1 \sin\alpha / T^4 - 14\,a_1\,c_1\,b_1^2 \sin\alpha . \cos\alpha / T^4 + 24\,a_1^2\,b_1^2\,c_1 \sin\alpha . \cos\alpha (a_1 + b_1 \cos\alpha) / T^6 - 24\,a_1^2\,b_1^3\,c_1 \sin^3\alpha / T^6 + 48\,a_1^3\,b_1^3\,c_1\,(a_1 + b_1 \cos\alpha) / T^8 \qquad (2.31)$$

Numerical results have been obtained for the changes in acceleration and jerk due to variations in a, b and c for an internal Geneva mechanism having 4 slots with a crank length of 10 cm. The results are given in Tables 2.2 to 2.5. A summary of the maximum percent changes in acceleration and jerk obtainable for various values of Δ a, Δ b and Δ c is given in Table 2.6. It can be observed that contrary to the results of the external Geneva mechanism, here the effect of Δ c (that is the clearance in the slot width) on the acceleration is greater than the effects of Δ a and Δ b.

2.4 PROBABILISTIC ERROR ANALYSIS CONSIDERING TOLERANCES AND CLEARANCES FOR THE EXTERNAL GENEVA MECHANISM

Figure 2.9 shows a probabilistic model of the external Geneva mechanism by considering both tolerances and clearances as random variables. Some clearance between the pin of one link and the race of the adjacent link is always present in practice. Here r_{12} is the radial clearance between the output wheel (follower) and its shaft and r_{31} is the radial clearance between the input wheel (crank) and its shaft. Obviously, the clearance zone between the roller and the slot is rectangular. r_{23} shows the width of this rectangular clearance zone. A rectangular coordinate system (x_{ij}, y_{ij}) is so chosen that the x_{ij} axis is coincident with the centerline of the link ending in the race. For most of the practical problems it can be safely assumed that the probability of the pin axis lying at any point is same for all the points inside the clearance zones. The equivalent linkage is shown by dotted lines in Fig. 2.9. From the geometry of this figure, it can be seen that

$$
\begin{aligned}
a_e^2 &= (a_1 + x_{12})^2 + y_{12}^2 \\
b_e^2 &= (b_1 + x_{31})^2 + y_{31}^2 \qquad (2.32) \\
c_e &= y_{23}
\end{aligned}
$$

Since $(a_1 + x_{12}) \ggg y_{12}$ and $(b_1 + x_{31}) \ggg y_{31}$, hence $a_e \simeq a_1 + x_{12}$ and $b_e \simeq b_1 + x_{31}$. In the previous section, the expressions for acceleration and

jerk have been derived as

$$\begin{aligned} \text{Acceleration} &= A\,(a_1,\, b_1,\, c_1, \alpha) \\ \text{Jerk} &= J\,(a_1,\, b_1,\, c_1, \alpha) \end{aligned} \tag{2.33}$$

By considering the equivalent linkage with tolerances and clearances, the new expressions for acceleration and jerk can be written as

$$\begin{aligned} \text{Acceleration} &= A\,(a_e,\, b_e,\, c_e, \alpha) \\ \text{Jerk} &= J\,(a_e,\, b_e,\, c_e, \alpha) \end{aligned} \tag{2.34}$$

i.e.,

$$\begin{aligned} \text{Acceleration} &= A\,(a + \Delta a + x_{12},\, b + \Delta b + x_{31},\, y_{23}, \alpha) \\ \text{Jerk} &= J\,(a + \Delta a + x_{12},\, b + \Delta b + x_{31},\, y_{23}, \alpha) \end{aligned} \tag{2.35}$$

In the above expressions, Δa, Δb, x_{12}, y_{23} and x_{31} are random variables.

If x_{ij}, y_{ij} are the coordinates of the pin axis, the probability density function is given by

$$f\,(x_{12},\, y_{12}) = \begin{cases} \dfrac{1}{\pi r_{12}^2} & , \quad x_{12}^2 + y_{12}^2 \leq r_{12}^2 \\ 0 & , \quad x_{12}^2 + y_{12}^2 > r_{12}^2 \end{cases} \tag{2.36}$$

$$f\,(x_{31},\, y_{31}) = \begin{cases} \dfrac{1}{\pi r_{31}^2} & , \quad x_{31}^2 + y_{31}^2 \leq r_{31}^2 \\ 0 & , \quad x_{31}^2 + y_{31}^2 > r_{31}^2 \end{cases} \tag{2.37}$$

The upper bound on the length of the rectangular clearance zone is $(2\ r_{12} + 2\ r_{31})$. Hence

$$f\ (x_{23}, y_{23}) = \begin{cases} \dfrac{1}{2(r_{12} + r_{31})\ 2.r_{23}}\ , & \{(-2\ r_{12} \leqslant x_{23} \leqslant 2\ r_{31}) \cap (-r_{23} \leqslant y_{23} \leqslant r_{23}) \\ 0\ , & \text{otherwise} \end{cases} \quad (2.38)$$

If X_1 and X_2 are two random variables, then the correlation moment of X_1 and X_2 is defined as

$$K_{X_1\ X_2} = \iint x_1\ x_2\ f\ (x_1, x_2)\ dx_1\ dx_2 \quad (2.39)$$

Hence

$$K_{x_{12}\ y_{12}} = \int_{-r_{12}}^{r_{12}} dy_{12} \int_{-(r_{12}^2 - y_{12}^2)^{1/2}}^{(r_{12}^2 - y^2)^{1/2}} x_{12}\ y_{12} \frac{1}{\pi(x_{12}^2 + y_{12}^2)}\ dx_{12}$$

$$= \quad 0.$$

Similarly it can be proved that $K_{x_{31}\ y_{31}} = 0$.

$$K_{x_{23}\ y_{23}} = \int_{r_{23}}^{-r_{23}} \int_{-2\ r_{12}}^{2\ r_{31}} x_{23}\ y_{23} \frac{1}{(2\ r_{12} + 2\ r_{31})2.\ r_{23}}\ dx_{23}\ dy_{23}$$

$$= \quad r_{23}\ (r_{12} - r_{31}) \quad (2.40)$$

It is assumed that the shift of the roller is approximately same on both the sides from the mean position and hence $K_{x_{23}\ y_{23}}$ is approximately zero. Hence, the random variables x_{ij} and y_{ij} can be assumed to be uncorrelated.

The mean and standard deviations of the various random variables can be derived as follows |7|:

The marginal density function, $f_{Y_{23}}(y_{23})$, is given by

$$f_{Y_{23}}(y_{23}) = \int_{-2\,r_{12}}^{2\,r_{31}} \frac{1}{2\,(r_{12}+r_{31})\,2\,r_{23}}\,dx_{23}\,, \tag{2.42}$$

$$\mu_{y_{23}} = \int_{-r_{23}}^{r_{23}} y_{23}\,\frac{1}{2\,r_{23}}\,dy_{23} = 0\,, \tag{2.43}$$

and

$$\sigma^2_{y_{23}} = \int_{-r_{23}}^{r_{23}} y^2_{23}\,\frac{1}{2\,r_{23}}\,dy_{23} = \frac{r^2_{23}}{3} \tag{2.44}$$

Considering the tolerances on the link lengths as normally distributed (with $\pm 3\sigma$ band), the mean and standard deviations of the various random variables are given below:

Random Variable	Δa	Δb	x_{12}	x_{31}	y_{23}
Mean	0	0	0	0	0
Variance	$(\Delta a/3)^2$	$(\Delta b/3)^2$	$x^2_{12}/4$	$x^2_{31}/4$	$y^2_{23}/3$

From Eqs. (2.35) it can be observed that the acceleration and the jerk are functions of the random variables Δa, Δb, x_{12}, y_{32} and x_{31}. Using the partial derivative rule, the standard deviations of the acceleration and the jerk can be computed as follows:

$$(\sigma_A)_j^2 = \left[\sum_{i=1}^{5} \left(\frac{\partial A_j}{\partial x_i} \right)^2 \Bigg|_{\vec{\underline{X}}} \cdot (\sigma x_i)^2 \right] \tag{2.13}$$

$$(\sigma_J)_j^2 = \sum_{i=1}^{5} \left(\frac{\partial J_j}{\partial x_i} \right)^2 \Bigg|_{\vec{\underline{X}}} (\sigma x_i)^2 \tag{2.14}$$

where $(\sigma_A)_j$ = standard deviation of acceleration at $\alpha = \alpha_j$,

$(\sigma_J)_j$ = standard deviation of jerk at $\alpha = \alpha_j$,

A_j = Acceleration at $\alpha = \alpha_j$, J_j = Jerk at $\alpha = \alpha_j$

$\vec{\underline{X}}$ = mean design vector = Null vector

= $(0, 0, 0, 0, 0)^T$.

The partial derivatives, required for the computation of standard deviation of acceleration and jerk, have been derived in closed form and are given in Appendix A.

Optimum Allocation of Tolerances and Clearances:

Very small values of tolerances and clearance, though give rise to small deviation in the acceleration and jerk, result in high manufacturing cost. Hence, maximum possible values of tolerance and clearance should be allocated to various link lengths, keeping the deviation of the acceleration and jerk within tolerable limits. This problem can be formulated as an optimization problem as follows:

Find
$$X = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{Bmatrix} = \begin{Bmatrix} \Delta a \\ \Delta b \\ x_{12} \\ y_{23} \\ x_{21} \end{Bmatrix}$$

which minimizes
$$OBJ = \sum_{i=1}^{5} \frac{1}{x_i}$$

subject to the following constraints

$$l_i \leq x_i \leq U_i \quad , \quad i = 1 \text{ to } 5$$

$$(\sigma_A)_j \leq E_j$$

$$(\sigma_J)_j \leq F_j$$

where l_i and U_i denote the lower and the upper bounds on x_i ($i = 1$ to 5) and E_j and F_j represent the maximum permissible values of σ_A and σ_J at $\alpha = \alpha_j$. The objective function, OBJ, represents a measure of the manufacturing cost. The constraint on the standard deviation of acceleration has been incorporated to limit the magnitude of inertia forces. In a similar manner, to ensure smooth operation of the mechanism, the rate of change of acceleration (that is jerk) has also been controlled by placing an upper bound on the standard deviation of the jerk.

The nonlinear programming techniques (interior penalty function method coupled with variable metric method of unconstrained minimization and cubic interpolation method of one dimensional minimization) have been used to solve this problem |6| .

NUMERICAL EXAMPLE:

The data of the numerical example considered is given below.

Upper bounds on the design variables:

U_i = (3% a, 4% b, 0.2, 0.2, 0.2)

Lower bounds on the design variables:

l_i = (0.2% a, 0.2% b, 0.01, 0.01, 0.01)

Range of the input angle for a 4 slot mechanism with crank length of 10 cm is from -45° to $+45^\circ$. The standard deviations of the acceleration and the jerk have been calculated at 5° interval. The values of E_j and F_j are taken as:

E_j = 20% Mean acceleration at $\alpha = \alpha_j$,

F_j = 30% Mean jerk at $\alpha = \alpha_j$.

The results of optimization are shown in Table 2.7. There has been a reduction of 35.2% in the objective function. The upper bounds on the standard deviation of jerk at $\alpha = 0^\circ$ and $\alpha = 5^\circ$ have been found to be active at the optimum point. The reduction in the objective function with the number of one dimensional minimization steps is shown in Fig. 2.10. The $\pm 3\sigma$ band widths of acceleration and jerk corresponding to the starting design vector are given in Table 2.6. The band width corresponding to the optimum point are shown graphically in Figs. 2.11 and 2.12. The band widths can be seen to be large at the

central part compared to those at the ends. The band widths are larger at the optimum point compared to those at the starting point. This reflects increase in tolerances and clearances at the optimum point compared to those at the initial point.

2.5 PROBABILISTIC ERROR ANALYSIS CONSIDERING TOLERANCES AND CLEARANCES FOR THE INTERNAL GENEVA MECHANISM

The probabilistic model of the internal Geneva mechanism is similar to that of the external Geneva mechanism. The Eqs. (2.13) and (2.14), for the standard deviation of acceleration and jerk, apply here also. The partial derivatives, required for the computation of standard deviations, are given in Appendix A. The formulation of the optimization problem is the same as that for the external Geneva mechanism.

NUMERICAL EXAMPLE:

The data of the numerical example considered is given below.

Upper bounds on the design variables:

U_i = (3% a, 4% b, 0.2, 0.2, 0.2)

Lower bounds on the design variables:

l_i = (0.2% a, 0.2% b, 0.01, 0.01, 0.01)

Range of the input angle for a 4 slot mechanism with crank length of 10 cm. is from -135° to

$+ 135^{\circ}$. The standard deviations of acceleration and jerk have been calculated at 5° interval and the upper bounds on the standard deviation of acceleration and jerk are taken as $E_j = 0.05$ and $F_j = 0.4$ for all j's.

In this case it is not feasible to take E_j and F_j as fixed percentages of the mean acceleration and the mean jerk respectively (as has been done for external Geneva mechanism) because the mean values of acceleration and jerk are extremely small.

The results of optimization are summarized in Table 2.9. It can be seen that the objective function has been reduced by 91.3%. The upper bounds on the design variables Δa, Δb, x_{12}, and x_{31} have been found to be active at the optimum point. Among the behaviour constraints, the upper bound on the standard deviation of jerk at $\alpha = 135^{\circ}$ has been found to be critical at the final point. The progress of optimization with respect to the number of one dimensional steps is shown in Fig. 2.13. The $\pm 3\sigma$ band widths of acceleration and jerk are given in Table 2.10 for the starting design vector and are shown graphically in Figs. 2.14 and 2.15 for the optimum design vector. It can be seen that the band widths are small at the middle compared to those at the starting and ending angles. The band widths can be seen to be much larger at the optimum point compared to those at the starting point. This is to be expected since the starting design

vector is nearer to the lower bound vector while the optimum design vector is closer to the upper bound vector.

DISCUSSION OF THE RESULTS:

The optimization programmes have been run on DEC 1090 system (a fourth generation computer). The optimization problem for the external Geneva mechanism took 1.22 (1 min. and 22 sec.) CPU (Central Processing Unit) time while for the internal Geneva mechanism problem the time taken was 1.38. From Tables 2.7 and 2.9, it can be concluded that the objective function decreases rapidly in the first few iterations. If computer time is also a criteria then it is sufficient to optimize for two values of the penalty parameter r_k. For the subsequent values of r_k more than two, the decrease in the value of the objective function is not significant. Another interesting result is that for both the mechanisms, the active constraints at the optimum point correspond to upper bounds on some of the design variables and upper bounds on the standard deviation of jerk only. The standard deviations of the acceleration at various crank angles are well within the limits at the optimum point.

TABLE - 2.1

SUMMARY OF NUMERICAL RESULTS OF DETERMINISTIC ERROR ANALYSIS OF EXTERNAL GENEVA MECHANISM.

Maximum % change in the jerk				
Variables	+ 1% of mean	- 1% of mean	+ 3% of mean	- 3% of mean
Δ a	- 15.3, + 11.6	- 15.0, + 17.2	+ 27.1, - 40.9	- 58.2, + 58.2
Δ b	- 14.8, 17.0	11.7, - 15.4	56.3, - 55.9	27.6, - 42.0
Δ c	- 2.3, 2.3	+ 2.3, - 2.3	- 7.1, 7.12	+ 7.1 , - 7.1
Maximum % change in the acceleration				
Variables	+ 1% of mean	- 1% of mean	+ 3% of mean	- 3% of mean
Δ a	- 7.5, 0.94	8.4, - 1.06	- 20.3, 2.4	- 3.5, 28.6
Δ b	8.3, - 1.0	- 7.6, 0.94	27.6, - 3.4	2.5, - 20.8
Δ c	- 0.56, .56	0.56, - 0.56	1.71, - 1.6	1.71, - 1.6

TABLE 2.2

Angle ϕ_2	% change in acceleration due to			% change in jerk due to		
	$\Delta a = -1\% a$	$\Delta b = -1\% b$	$\Delta c = -1\% c$	$\Delta a = -1\% a$	$\Delta b = -1\% b$	$\Delta c = -1\% c$
-135	-0.10610E+01	0.94991E+00	-0.28324E+00	0.25792E+00	-0.40941E+00	-0.23584E+00
-130	-0.13548E+01	0.12515E+01	-0.29423E+00	-0.28450E+00	0.14059E+00	-0.25479E+00
-125	-0.16011E+01	0.15112E+01	-0.30300E+00	-0.70177E+00	0.55935E+00	-0.26882E+00
-120	-0.17878E+01	0.17073E+01	-0.31032E+00	-0.10291E+01	0.90777E+00	-0.27938E+00
-115	-0.19377E+01	0.18552E+01	-0.31572E+00	-0.12902E+01	0.11802E+01	-0.28739E+00
-110	-0.20585E+01	0.19938E+01	-0.32260E+00	-0.15017E+01	0.14021E+01	-0.29343E+00
-105	-0.21578E+01	0.20996E+01	-0.32828E+00	-0.16753E+01	0.15851E+01	-0.29788E+00
-100	-0.22402E+01	0.21874E+01	-0.33405E+00	-0.18195E+01	0.17379E+01	-0.30099E+00
-95	-0.23088E+01	0.22610E+01	-0.34016E+00	-0.19407E+01	0.18657E+01	-0.30291E+00
-90	-0.23668E+01	0.23231E+01	-0.34639E+00	-0.20435E+01	0.19753E+01	-0.30368E+00
-85	-0.24159E+01	0.23758E+01	-0.35418E+00	-0.21318E+01	0.20705E+01	-0.30332E+00
-80	-0.24577E+01	0.24207E+01	-0.36327E+00	-0.22080E+01	0.21520E+01	-0.30190E+00
-75	-0.24935E+01	0.24593E+01	-0.37362E+00	-0.22743E+01	0.22231E+01	-0.29901E+00
-70	-0.25243E+01	0.24924E+01	-0.38599E+00	-0.23325E+01	0.22857E+01	-0.29483E+00
-65	-0.25508E+01	0.25210E+01	-0.40095E+00	-0.23839E+01	0.23410E+01	-0.28910E+00
-60	-0.25735E+01	0.25456E+01	-0.41923E+00	-0.24296E+01	0.23901E+01	-0.28161E+00
-55	-0.25933E+01	0.25669E+01	-0.44186E+00	-0.24703E+01	0.24339E+01	-0.27215E+00
-50	-0.26103E+01	0.25853E+01	-0.47022E+00	-0.25066E+01	0.24732E+01	-0.26049E+00
-45	-0.26249E+01	0.26011E+01	-0.50630E+00	-0.25391E+01	0.25092E+01	-0.24635E+00
-40	-0.26374E+01	0.26146E+01	-0.55310E+00	-0.25680E+01	0.25395E+01	-0.22957E+00
-35	-0.26479E+01	0.26260E+01	-0.61528E+00	-0.25935E+01	0.25672E+01	-0.20994E+00
-30	-0.26558E+01	0.26356E+01	-0.70064E+00	-0.26159E+01	0.25913E+01	-0.18737E+00
-25	-0.26610E+01	0.26434E+01	-0.82315E+00	-0.26351E+01	0.26120E+01	-0.16186E+00
-20	-0.26698E+01	0.26497E+01	-0.10108E+01	-0.26509E+01	0.26292E+01	-0.13354E+00
-15	-0.26742E+01	0.26545E+01	-0.13288E+01	-0.26634E+01	0.26428E+01	-0.10271E+00
-10	-0.26773E+01	0.26579E+01	-0.13728E+01	-0.26725E+01	0.26526E+01	-0.69904E-01
-5	-0.26792E+01	0.26598E+01	-0.33208E+01	-0.26780E+01	0.26585E+01	-0.35432E-01

TABLE 2.3

Angle ϕ_2	% change in acceleration due to			% change in jerk due to		
	$\Delta a = 1\% a$	$\Delta b = 1\% b$	$\Delta c = 1\% c$	$\Delta a = 1\% a$	$\Delta b = 1\% b$	$\Delta c = 1\% c$
-135	0.94100E+00	-0.10499E+01	0.28244E+00	-0.40463E+00	0.25611E+00	0.23557E+00
-130	0.12524E+01	-0.13507E+01	0.29331E+00	0.13998E+00	-0.28093E+00	0.25424E+00
-125	0.14965E+01	-0.15848E+01	0.30202E+00	0.56381E+00	-0.69412E+00	0.26806E+00
-120	0.16903E+01	-0.17536E+01	0.30930E+00	0.88933E+00	-0.10182E+01	0.27847E+00
-115	0.18470E+01	-0.19176E+01	0.31558E+00	0.11590E+01	-0.12759E+01	0.28637E+00
-110	0.19743E+01	-0.20377E+01	0.32156E+00	0.13985E+01	-0.14852E+01	0.29234E+00
-105	0.20790E+01	-0.21361E+01	0.32726E+00	0.15698E+01	-0.15581E+01	0.29675E+00
-100	0.21559E+01	-0.22176E+01	0.33305E+00	0.17210E+01	-0.18010E+01	0.29985E+00
-95	0.22388E+01	-0.22857E+01	0.33920E+00	0.18484E+01	-0.19211E+01	0.30175E+00
-90	0.23002E+01	-0.23431E+01	0.34594E+00	0.19570E+01	-0.20230E+01	0.30254E+00
-85	0.23523E+01	-0.23917E+01	0.35358E+00	0.20502E+01	-0.21103E+01	0.30221E+00
-80	0.23958E+01	-0.24331E+01 +1	0.36240E+00	0.21309E+01	-0.21857E+01	0.30071E+00
-75	0.24350E+01	-0.24686E+01	0.37279E+00	0.22013E+01	-0.22515E+01	0.29795E+00
-70	0.24678E+01	-0.24990E+01	0.38519E+00	0.22632E+01	-0.23091E+01	0.29382E+00
-65	0.24960E+01	-0.25253E+01	0.40018E+00	0.23179E+01	-0.23630E+01	0.28812E+00
-60	0.25205E+01	-0.25479E+01	0.41849E+00	0.23665E+01	-0.24057E+01	0.28068E+00
-55	0.25415E+01	-0.25574E+01	0.44115E+00	0.24099E+01	-0.24455E+01	0.27127E+00
-50	0.25597E+01	-0.25842E+01	0.46952E+00	0.24487E+01	-0.24815E+01	0.25963E+00
-45	0.25753E+01	-0.25987E+01	0.50563E+00	0.24834E+01	-0.25137E+01	0.24555E+00
-40	0.25887E+01	-0.26110E+01	0.55214E+00	0.25144E+01	-0.25423E+01	0.22881E+00
-35	0.26000E+01	-0.26215E+01	0.61464E+00	0.25417E+01	-0.25676E+01	0.20922E+00
-30	0.26094E+01	-0.26302E+01	0.70001E+00	0.25557E+01	-0.25898E+01	0.18658E+00
-25	0.26177E+01	-0.26374E+01	0.82254E+00	0.25861E+01	-0.26087E+01	0.16120E+00
-20	0.26234E+01	-0.26432E+01	0.40102E+01	0.25932E+01	-0.26245E+01	0.13291E+00
-15	0.26292E+01	-0.26475E+01	0.43282E+01	0.26165E+01	-0.25368E+01	0.10210E+00
-10	0.26315E+01	-0.26506E+01	0.43722E+01	0.26253E+01	-0.25458E+01	0.69212E-01
-5	0.26334E+01	-0.26525E+01	0.39203E+01	0.26321E+01	-0.25513E+01	0.34846E-01

TABLE 2.4

Angle ϕ_2	% change in acceleration due to $\Delta a = 5\%\,a$	$\Delta b = 5\%\,b$	$\Delta c = 5\%\,c$	% change in jerk due to $\Delta a = 5\%\,a$	$\Delta b = 5\%\,b$	$\Delta c = 5\%\,c$
-135	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
-130	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
-125	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
-120	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
-115	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
-110	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
-105	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
-100	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
-95	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
-90	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
-85	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
-80	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
-75	[illegible]	-0.12113E+02	0.11557E+01	[illegible]	[illegible]	[illegible]
-70	0.11791E+02	-0.12511E+02	0.13187E+01	0.10515E+02	-0.11717E+02	0.11591E+01
-65	0.11917E+02	-0.12650E+02	[illegible]	[illegible]	[illegible]	[illegible]
-60	0.12026E+02	-0.12715E+02	0.22351E+01	[illegible]	[illegible]	[illegible]
-55	0.12201E+02	-0.12826E+02	0.21226E+01	[illegible]	-0.12310E+02	0.13372E+01
-50	0.12307E+02	[illegible]	0.23198E+01	[illegible]	[illegible]	[illegible]
-45	0.12395E+02	-0.12956E+02	0.25215E+01	0.11877E+02	[illegible]	0.12109E+01
-40	0.12471E+02	-0.13007E+02	0.27558E+01	0.12051E+02	[illegible]	0.11365E+01
-35	0.12531E+02	-0.13051E+02	0.30563E+01	0.12215E+02	[illegible]	0.10380E+01
-30	0.12587E+02	-0.13097E+02	0.34393E+01	0.12310E+02	-0.13113E+02	0.10559E+01
-25	0.12631E+02	-0.13117E+02	0.41067E+01	0.12456E+02	[illegible]	0.72017E+00
-20	0.12665E+02	-0.13140E+02	0.53151E+01	0.12552E+02	-0.13053E+02	[illegible]
-15	0.12693E+02	-0.13150E+02	0.65351E+01	0.12597E+02	-0.13111E+02	0.50143E+00
-10	0.12712E+02	-0.13171E+02	0.78551E+01	[illegible]	-0.13151E+02	0.31015E+00
-5	0.12723E+02	-0.13173E+02	[illegible]	0.12715E+02	-0.13171E+02	0.15842E+00

TABLE 2.5

Angle ϕ_i	% change in acceleration due to $\Delta a = -5\% a$	$\Delta b = -5\% b$	$\Delta c = -5\% c$	% change in jerk due to $\Delta a = -5\% a$	$\Delta b = -5\% b$	$\Delta c = -5\% c$
-135	-0.66258E+01	0.37431E+01	-0.14211E+01	-0.13630E+00	-0.35008E+01	-0.11820E+01
-130	-0.80332E+01	0.54361E+01	-0.13915E+01	-0.30503E+01	-0.67136E+00	-0.12796E+01
-125	-0.91213E+01	0.67873E+01	-0.14525E+01	-0.50346E+01	0.15857E+01	-0.13519E+01
-120	-0.99617E+01	0.78745E+01	-0.15521E+01	-0.65553E+01	0.31097E+01	-0.14062E+01
-115	-0.10633E+02	0.87631E+01	-0.15942E+01	-0.77514E+01	0.42959E+01	-0.14473E+01
-110	-0.11171E+02	0.94916E+01	-0.16235E+01	-0.87291E+01	0.61250E+01	-0.14783E+01
-105	-0.11603E+02	0.10096E+02	-0.16518E+01	-0.94889E+01	0.71490E+01	-0.15009E+01
-100	-0.11970E+02	0.10600E+02	-0.16803E+01	-0.10130E+02	0.80179E+01	-0.15167E+01
-95	-0.12271E+02	0.11025E+02	-0.17106E+01	-0.10665E+02	0.87430E+01	-0.15267E+01
-90	-0.12523E+02	0.11385E+02	-0.17439E+01	-0.11119E+02	0.93772E+01	-0.15300E+01
-85	-0.12735E+02	0.11691E+02	-0.17815E+01	-0.11505E+02	0.99117E+01	-0.15279E+01
-80	-0.12916E+02	0.11954E+02	-0.18251E+01	-0.11837E+02	0.10383E+02	-0.15220E+01
-75	-0.13070E+02	0.12179E+02	-0.18765E+01	-0.12125E+02	0.10795E+02	-0.15057E+01
-70	-0.13203E+02	0.12374E+02	-0.19390E+01	-0.12373E+02	0.11159E+02	-0.14844E+01
-65	-0.13317E+02	0.12542E+02	-0.20125E+01	-0.12600E+02	0.11493E+02	-0.14553E+01
-60	-0.13415E+02	0.12687E+02	-0.21036E+01	-0.12797E+02	0.11771E+02	-0.14174E+01
-55	-0.13492E+02	0.12813E+02	-0.22155E+01	-0.12972E+02	0.12028E+02	-0.13696E+01
-50	-0.13572E+02	0.12921E+02	-0.23590E+01	-0.13128E+02	0.12259E+02	-0.13109E+01
-45	-0.13635E+02	0.13014E+02	-0.25382E+01	-0.13267E+02	0.12455E+02	-0.12398E+01
-40	-0.13698E+02	0.13094E+02	-0.27720E+01	-0.13391E+02	0.12650E+02	-0.11555E+01
-35	-0.13738E+02	0.13162E+02	-0.30928E+01	-0.13501E+02	0.12813E+02	-0.10570E+01
-30	-0.13771E+02	0.13219E+02	-0.35034E+01	-0.13596E+02	0.12955E+02	-0.94375E+00
-25	-0.13807E+02	0.13265E+02	-0.41210E+01	-0.13679E+02	0.13079E+02	-0.81588E+00
-20	-0.13825E+02	0.13302E+02	-0.50510E+01	-0.13746E+02	0.13181E+02	-0.67403E+00
-15	-0.13845E+02	0.13331E+02	-0.66499E+01	-0.13793E+02	0.13251E+02	-0.51964E+00
-10	-0.13853E+02	0.13351E+02	-0.98527E+01	-0.13833E+02	0.13319E+02	-0.35497E+00
-5	-0.13866E+02	0.13362E+02	-0.19610E+02	-0.13861E+02	0.13355E+02	-0.18300E+00

TABLE - 2.6

SUMMERY OF NUMERICAL RESULTS OF DETERMINISTIC ERROR ANALYSIS OF INTERNAL GENEVA MECHANISM.

Maximum % change in acceleration				
Variables	+ 1 % of mean	- 1 % of mean	+ 5 % of mean	- 5% of mean
Δ a	2.66 (Sym.)	- 2.67 (Sym.)	12.72 (Sym.)	- 13.866 (Sym.)
Δ b	-2.65 (Sym.)	+ 2.65 (Sym.)	-13.17 (Sym.)	13.36 (Sym.)
Δ c	3.92, -3.92(ASy.)	-3.92, 3.92(A Sym.)	+19.5, -19.6(ASy.)	-19.61, + 19.61

Maximum % change in jerk				
Variables	+ 1% of mean	- 1% of mean	+ 5% of mean	- 5% of mean
Δ a	2.63	- 2.67	- 3.25, 12.71	- 13.86
Δ b	- 2.65	2.65	-13.17	13.35
Δ c	.302, - .302	- 303, 303	+ 1.5, - 1.5	- 1.18, 1.18

TABLE 2.7

Starting feasible point $\vec{X}_1$ = (.022, .018, .015, .015, .015, .015)

Initial value of the penalty parameter r_1 = 1.0

Penalty function at the starting feasible point X_1 = PF = 354.8797

Objective function at the starting feasible point X_1 = OBJ = 301.0101

Number of iterations required for minimising PF = NITER

K	R_k	NITER	Optimum point X_k	PF	OBJ
1	$1.0\text{x}10^{0}$	10	(0.0283471, 0.0218191, 0.0196218, 0.0648426, 0.0153784)	268.757	212.520
2	$1.0\text{x}10^{-1}$	10	(0.0279130, 0.0220926, 0.0211337, 0.0853954, 0.0167284)	209.224	199.896
3	$1.0\text{x}10^{-2}$	10	(0.0288400, 0.0226918, 0.0216436, 0.0920816, 0.0166478)	198.247	195.873
4	$1.0\text{x}10^{-3}$	6	(0.0291842, 0.0227275, 0.0220552, 0.0920962, 0.0164711)	195.828	195.176
5	$1.0\text{x}10^{-4}$	3	(0.0291967, 0.0227480, 0.0220769, 0.0920974, 0.0165100)	195.146	194.934

GG(20) = −0.02818619 = Standard deviation on jerk at $\alpha = 5^\circ$

GG(29) = −0.006271601= Standard deviation on jerk at $\alpha = 0^\circ$

TABLE 2.8

Angle α	Standard deviation of A	Standard deviation of J	Mean acceleration	Mean jerk
-45	0.2603E-02	0.5044E-02	0.1000E+01	0.3000E+01
-40	0.2736E-02	0.9749E-02	0.1309E+01	0.4157E+01
-35	0.2900E-02	0.2197E-01	0.1738E+01	0.5775E+01
-30	0.4361E-02	0.4850E-01	0.2333E+01	0.7946E+01
-25	0.9691E-02	0.1003E+00	0.3136E+01	0.1045E+02
-20	0.2138E-01	0.1844E+00	0.4132E+01	0.1201E+02
-15	0.4086E-01	0.2616E+00	0.5098E+01	0.8830E+01
-10	0.6077E-01	0.1664E+00	0.5335E+01	-0.5830E+01
-5	0.5486E-01	0.3763E+00	0.3707E+01	-0.3235E+02
0	0.1876E-01	0.7465E+00	0.7159E-06	-0.4904E+02
5	0.5486E-01	0.3763E+00	-0.3707E+01	-0.3235E+02
10	0.6077E-01	0.1664E+00	-0.5335E+01	-0.5830E+01
15	0.4086E-01	0.2616E+00	-0.5098E+01	0.8830E+01
20	0.2138E-01	0.1844E+00	-0.4132E+01	0.1201E+02
25	0.9691E-02	0.1003E+00	-0.3136E+01	0.1045E+02
30	0.4361E-02	0.4850E-01	-0.2333E+01	0.7946E+01
35	0.2900E-02	0.2197E-01	-0.1738E+01	0.5775E+01
40	0.2736E-02	0.9749E-02	-0.1309E+01	0.4157E+01
45	0.2603E-02	0.5044E-02	-0.1000E+01	0.3000E+01

TABLE 2.9

Starting feasible point $\vec{X}_1$ = (.022, 0.018, 0.015, 0.015, 0.015)
Initial value of the penalty parameter r_1 = 1.0
Penalty function at the starting feasible point $\vec{X}_1$ = PF = 386.868
Objective function at the starting feasible point $\vec{X}_1$ = OBJ = 301.010

Number of iterations for minimizing PF = NITER

K	R_k	NITER	Optimum point X_k	PF at optimum	OBJ at optimum
1	1.0×10^{0}	9	(0.189846, 0174751, 0.135731, 0.0882011, 0.132750)	119.337	37.228
2	1.0×10^{-1}	10	(0.253663, 0241593, 0174373, 0.117303, 0.174177)	39.025	28.082
3	1.0×10^{-2}	10	(0.2802588, 0.267564, 0.190568, 0.126991, 0.190352)	27.419	25.651
4	1.0×10^{-3}	10	(0.292354, 0.279866, 0.195242, 0.130492, 0.194179)	25.292	24.928
5	1.0×10^{-4}	10	(0.294591, 0.277071, 0.199751, 0.130662, 0.198906)	24.816	24.690
6	1.0×10^{-5}	10	(0.294520, 0.277140, 0.199923, 0.131591, 0.199083)	24.660	24.627

The active constraints at the optimum point are as follows:

GG(1) = -0.018267 = Upper bound on Δa

GG(2) =- 0.020168 = Upper bound on Δb

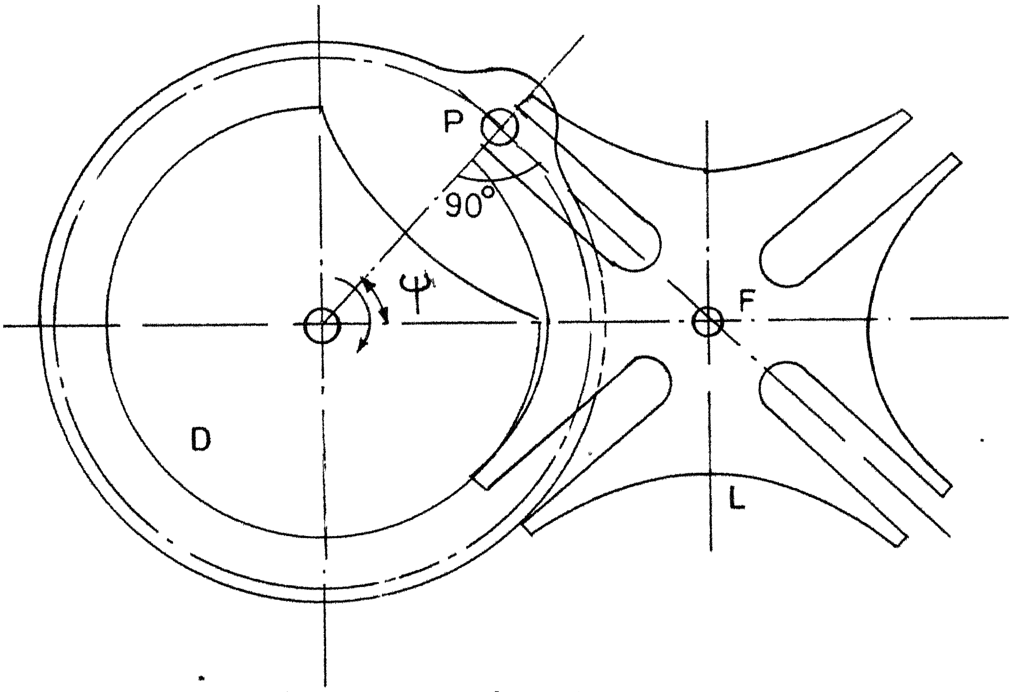
GG(3) =- 0.000382 = Upper bound on x_{12}

GG(5) =-.0244936 = Upper bound on x_{31}

GG(64)=-.0043934 = Standard deviation on jerk at $\alpha = 135^{\circ}$

TABLE 2.10

Angle α	standard deviation of A	standard deviation of J	mean acceleration	mean jerk
-135	0.26034E-02	0.34930E-01	0.10000E+01	-0.30000E+01
-130	0.23710E-02	0.21572E-01	0.77552E+00	-0.21926E+01
-125	0.20959E-02	0.13520E-01	0.61035E+00	-0.16256E+01
-120	0.18228E-02	0.86339E-02	0.48703E+00	-0.12234E+01
-115	0.15730E-02	0.56478E-02	0.39355E+00	-0.93453E+00
-110	0.13535E-02	0.38094E-02	0.32165E+00	-0.72411E+00
-105	0.11647E-02	0.26680E-02	0.26559E+00	-0.56869E+00
-100	0.10038E-02	0.19501E-02	0.22127E+00	-0.45231E+00
-95	0.86731E-03	0.14885E-02	0.18582E+00	-0.36405E+00
-90	0.75157E-03	0.11811E-02	0.15713E+00	-0.29630E+00
-85	0.65324E-03	0.96691E-03	0.13367E+00	-0.24372E+00
-80	0.56943E-03	0.81030E-03	0.11427E+00	-0.20249E+00
-75	0.49773E-03	0.69066E-03	0.98076E-01	-0.16987E+00
-70	0.43611E-03	0.59608E-03	0.84429E-01	-0.14385E+00
-65	0.38289E-03	0.51951E-03	0.72821E-01	-0.12295E+00
-60	0.33673E-03	0.45656E-03	0.62855E-01	-0.10605E+00
-55	0.29649E-03	0.40435E-03	0.54220E-01	-0.92320E-01
-50	0.26127E-03	0.36087E-03	0.46668E-01	-0.81133E-01
-45	0.23034E-03	0.32464E-03	0.40000E-01	-0.72000E-01
-40	0.20312E-03	0.29454E-03	0.34053E-01	-0.64548E-01
-35	0.17917E-03	0.26968E-03	0.28694E-01	-0.58489E-01
-30	0.15819E-03	0.24935E-03	0.23811E-01	-0.53600E-01
-25	0.13999E-03	0.23299E-03	0.19310E-01	-0.49708E-01
-20	0.12455E-03	0.22017E-03	0.15110E-01	-0.46681E-01
-15	0.11200E-03	0.21052E-03	0.11140E-01	-0.44421E-01
-10	0.10262E-03	0.20381E-03	0.73369E-02	-0.42855E-01
-5	0.96756E-04	0.19985E-03	0.36418E-02	-0.41934E-01
0	0.94757E-04	0.19854E-03	0.93052E-09	-0.41611E-01



P = Pin on the driving member D

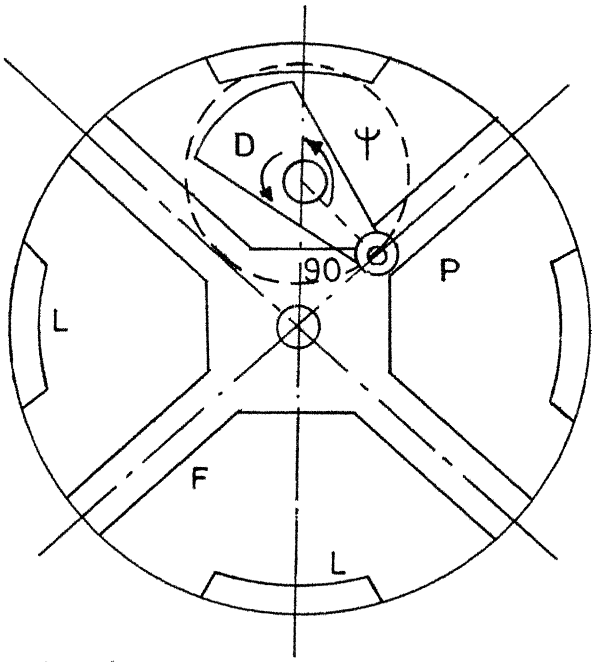
D = Driving wheel (input wheel)

F = Follower (output wheel)

L = Locking arm

2Ψ = Indexing period of the driver

FIG. 2.1.a EXTERNAL GENEVA WHEEL AT ENGAGEMENT



D = Driving wheel

P = Pin (roller) on the driving member

F = Follower

L = Locking arm

2ψ = Indexing period of the driver

FIG. 2.1.b INTERNAL GENEVA MECHANISM AT ENGAGEMENT

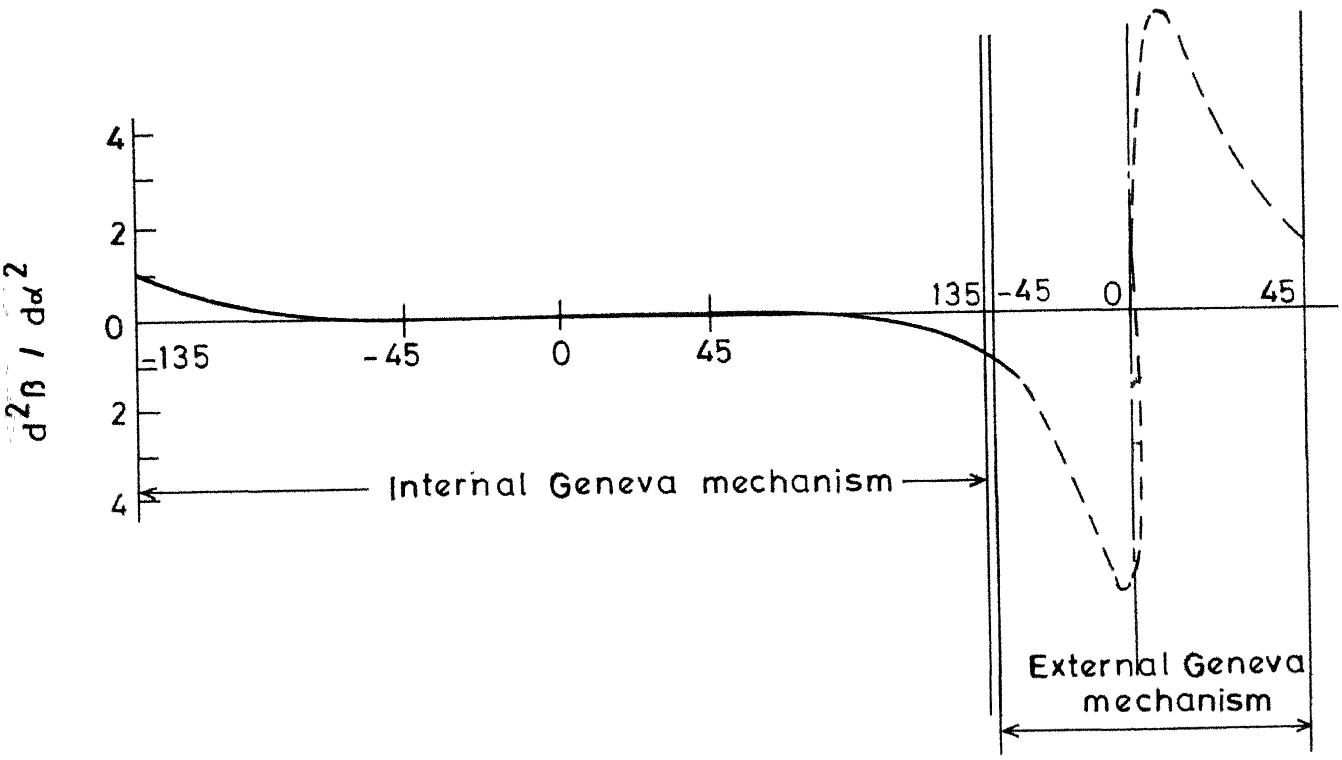


FIG. 2·1·c COMPARISON OF ACCELERATION IN EXTERNAL AND INTERNAL GENEVA MECHANISM

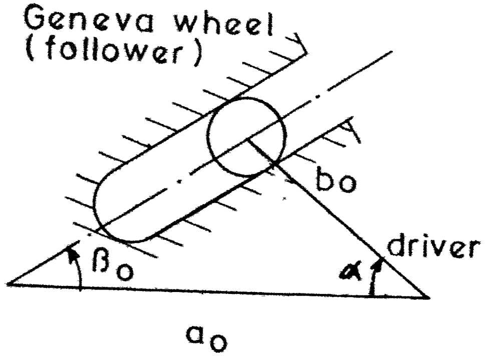


FIG. 2·2·a EXTERNAL GENEVA MECHANISM WITHOUT TOLERANCE

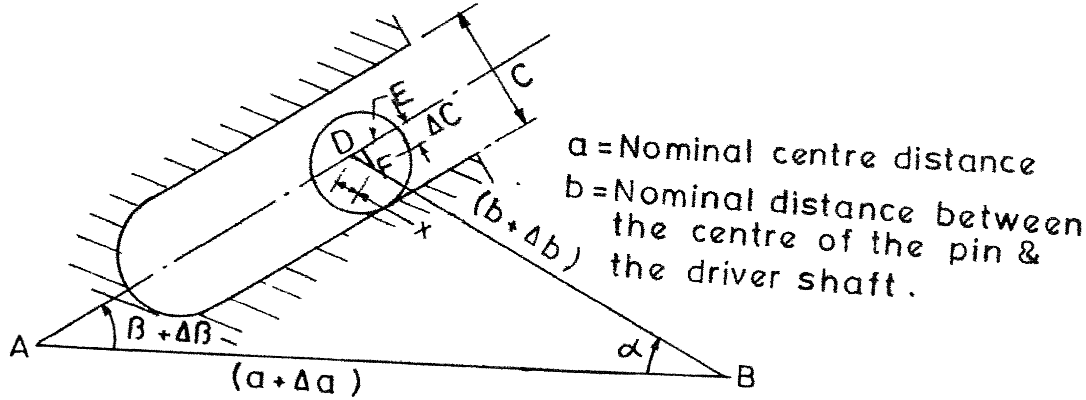
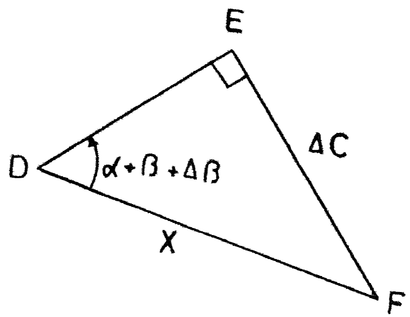


FIG. 2·2·b EXTERNAL GENEVA MECHANISM WITH TOLERANCES



GENEVA MECHANISM WITH TOLERANCES
(enlarged view of ΔDEF of fig.2·2b)

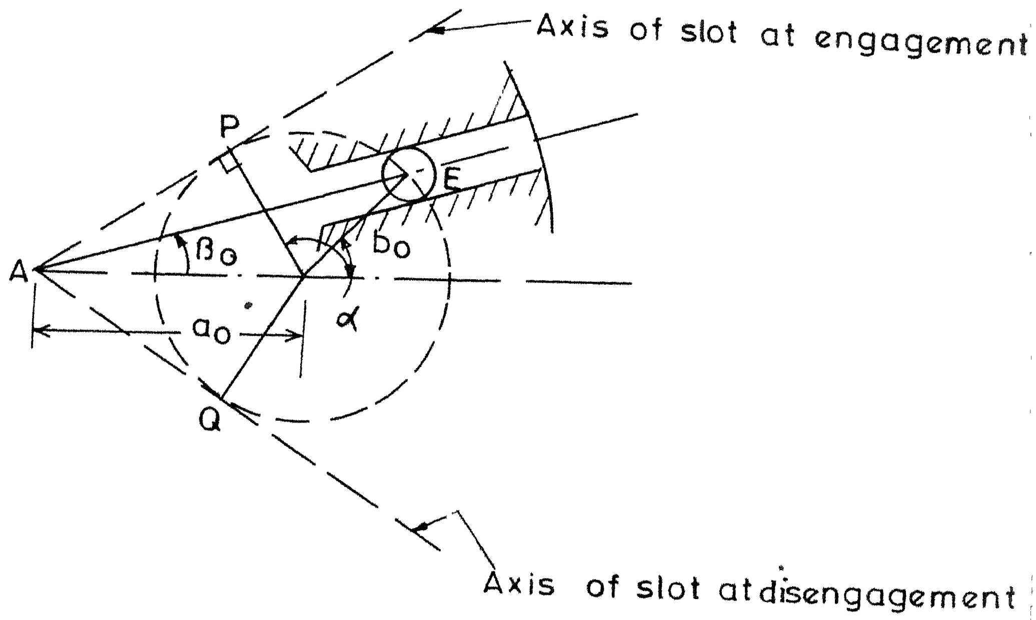


FIG. 2.2.c INTERNAL GENEVA MECHANISM WITHOUT TOLERANCE

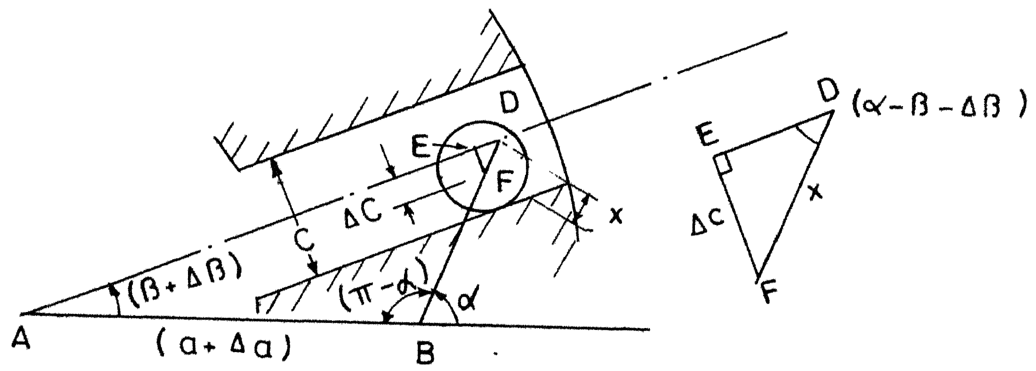


FIG. 2.2.d INTERNAL GENAVA MECHANISM WITH TOLERANCES

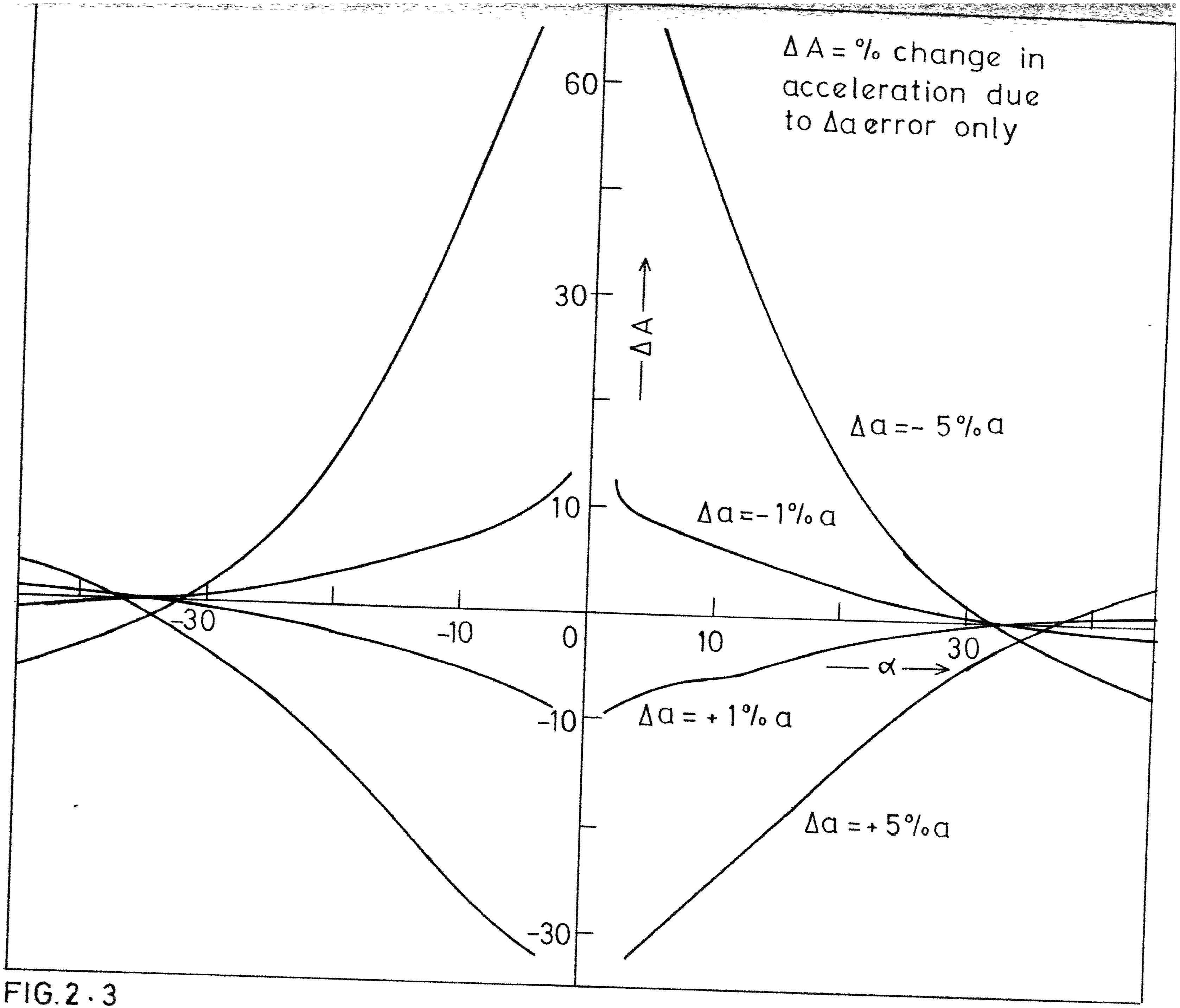


FIG. 2.3

DETERMINISTIC ERROR ANALYSIS OF EXTERNAL GENEVA

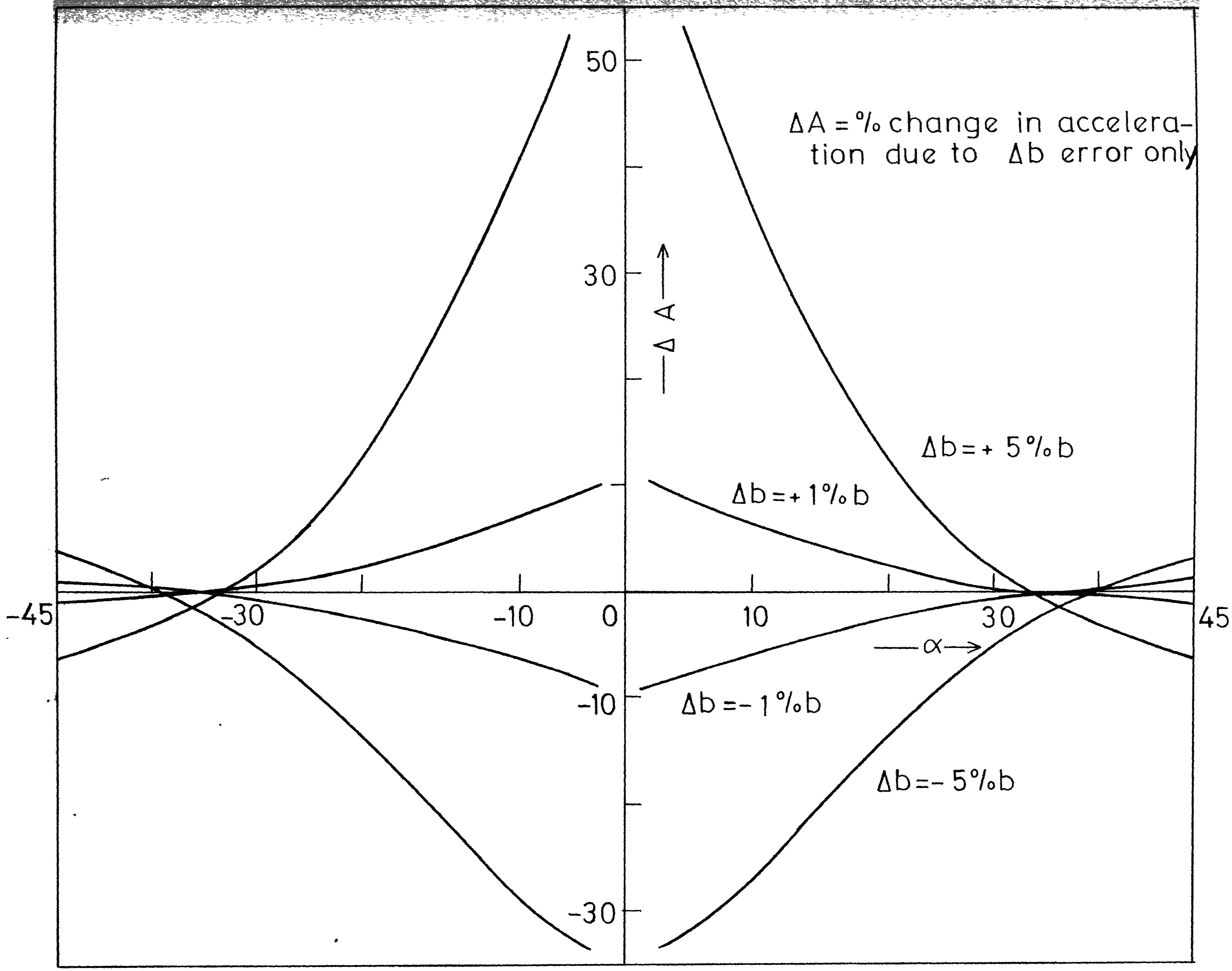
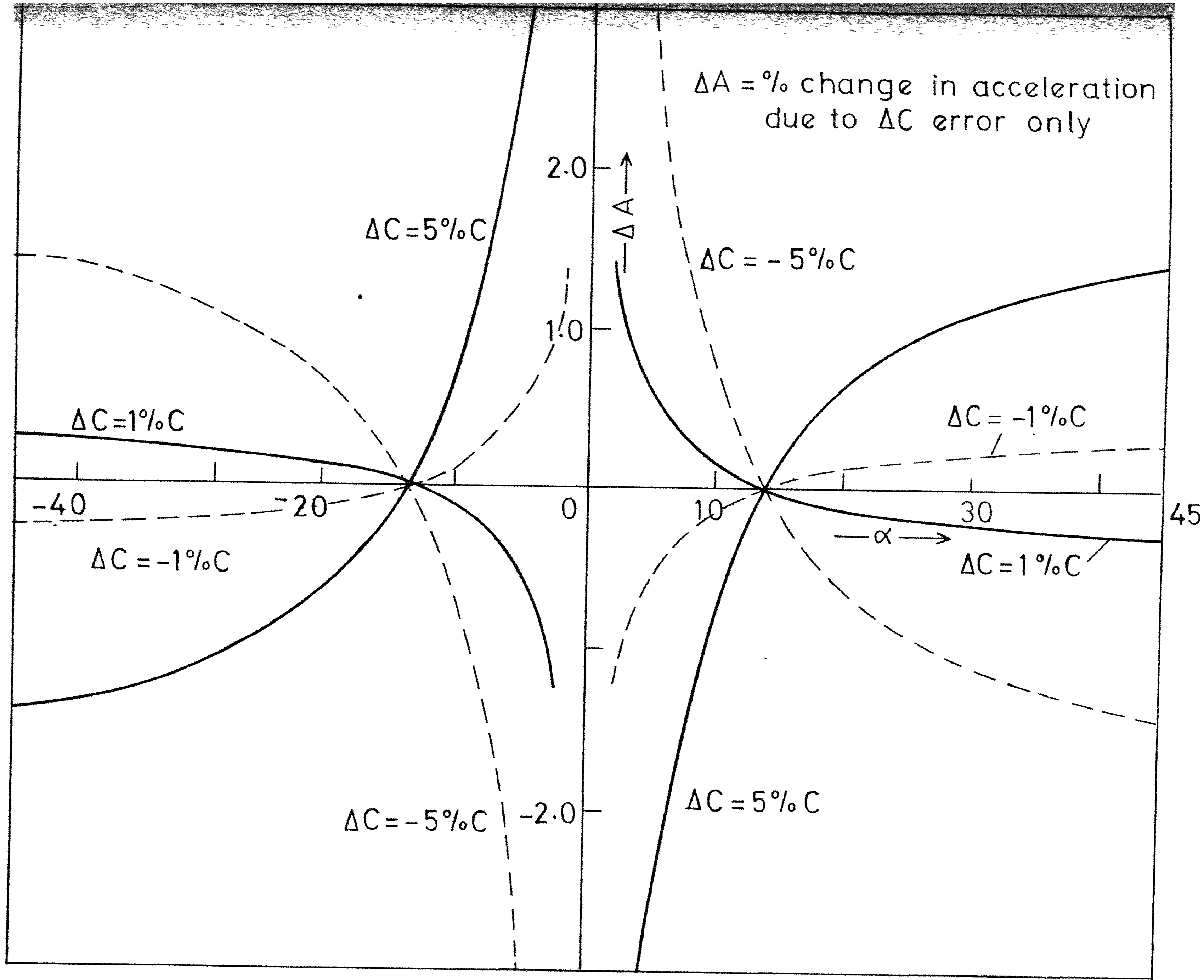


FIG. 2.4

DETERMINISTIC ERROR ANALYSIS OF EXTERNAL GENEVA MECHANISM



DETERMINISTIC ERROR ANALYSIS OF EXTERNAL GENEVA MECHANISM

FIG 2.5

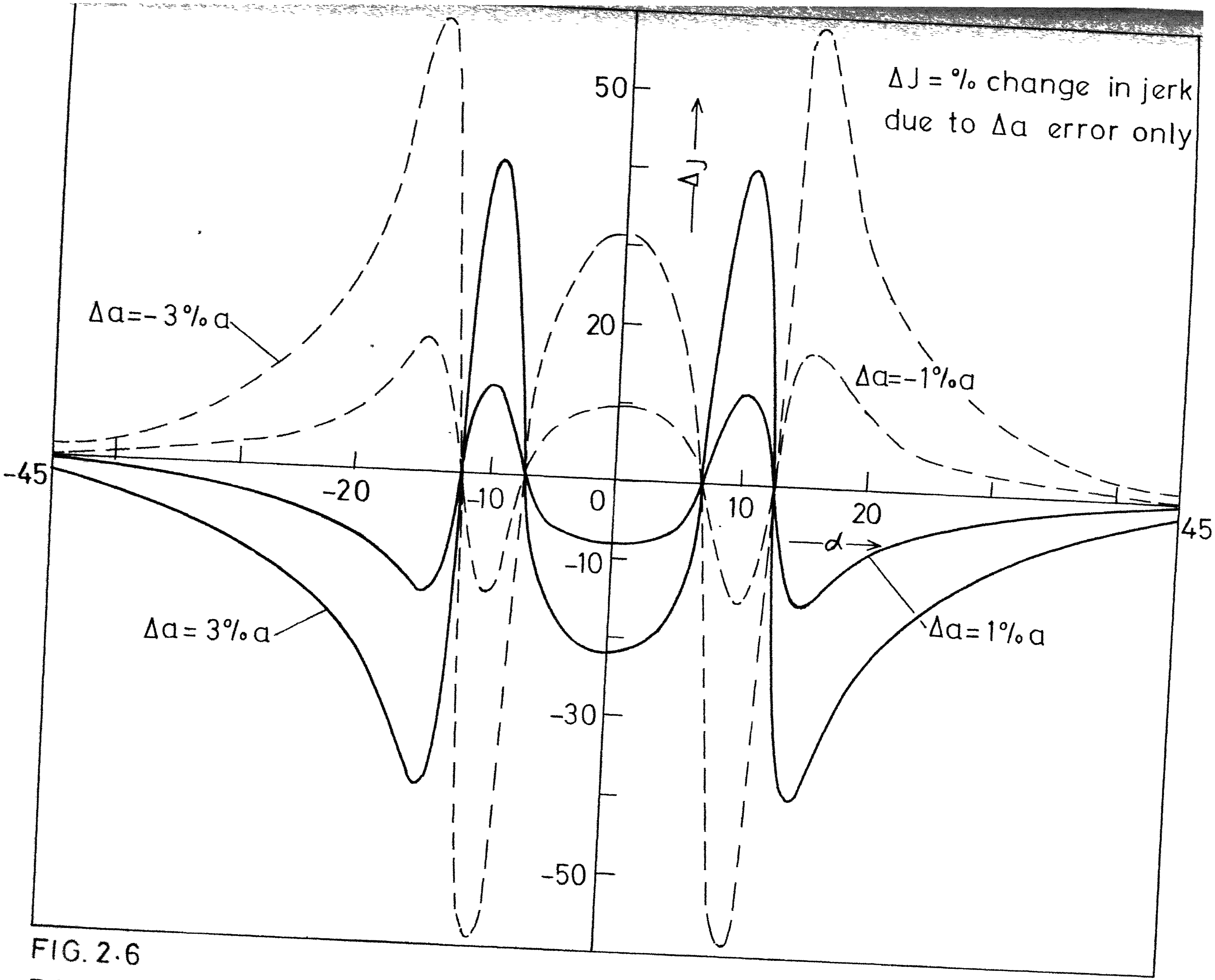


FIG. 2.6

DETERMINISTIC ERROR ANALYSIS OF EXTERNAL GENEVA MECHANISM

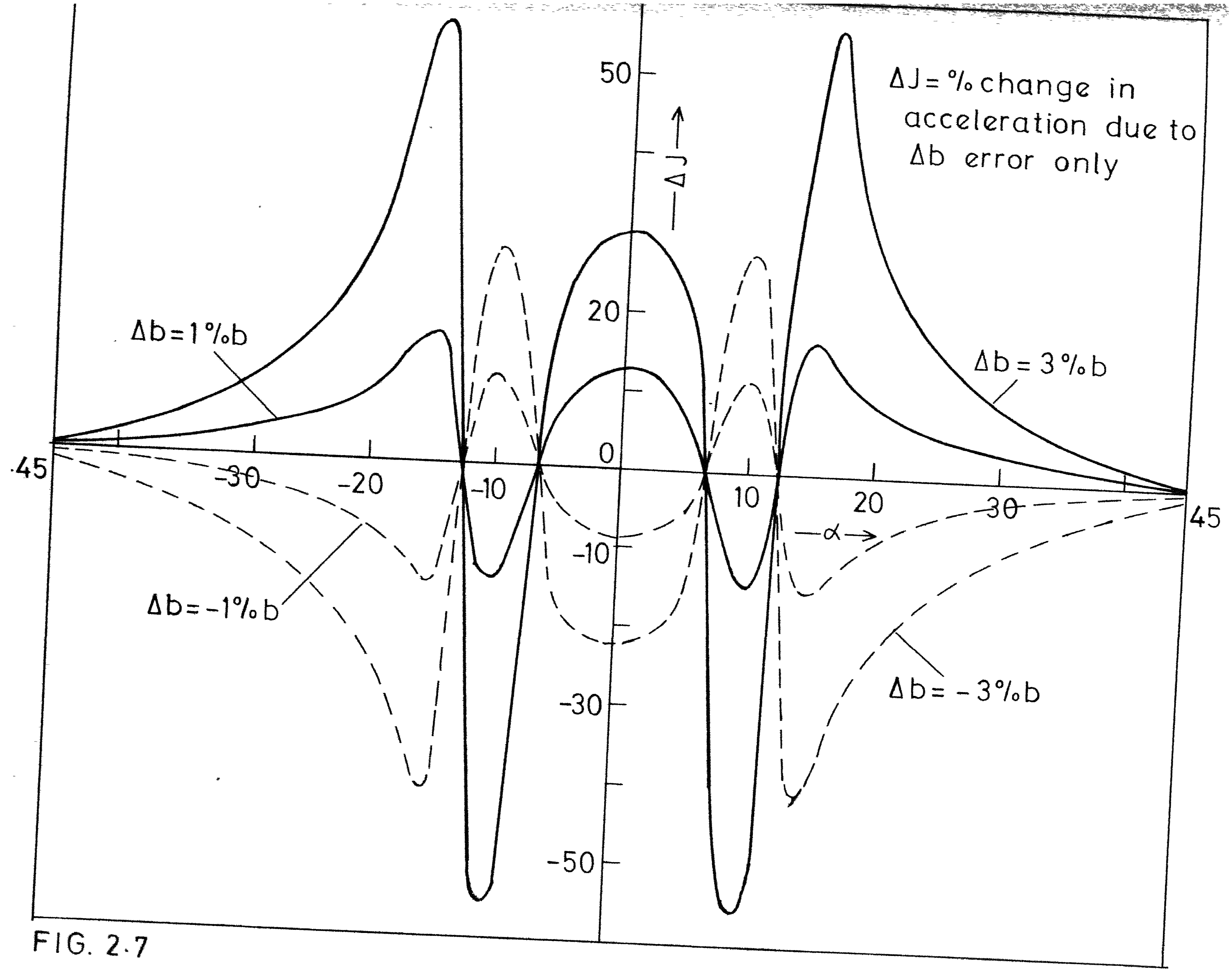


FIG. 2·7

DETERMINISTIC ERROR ANALYSIS OF EXTERNAL GENEVA MECHANISM

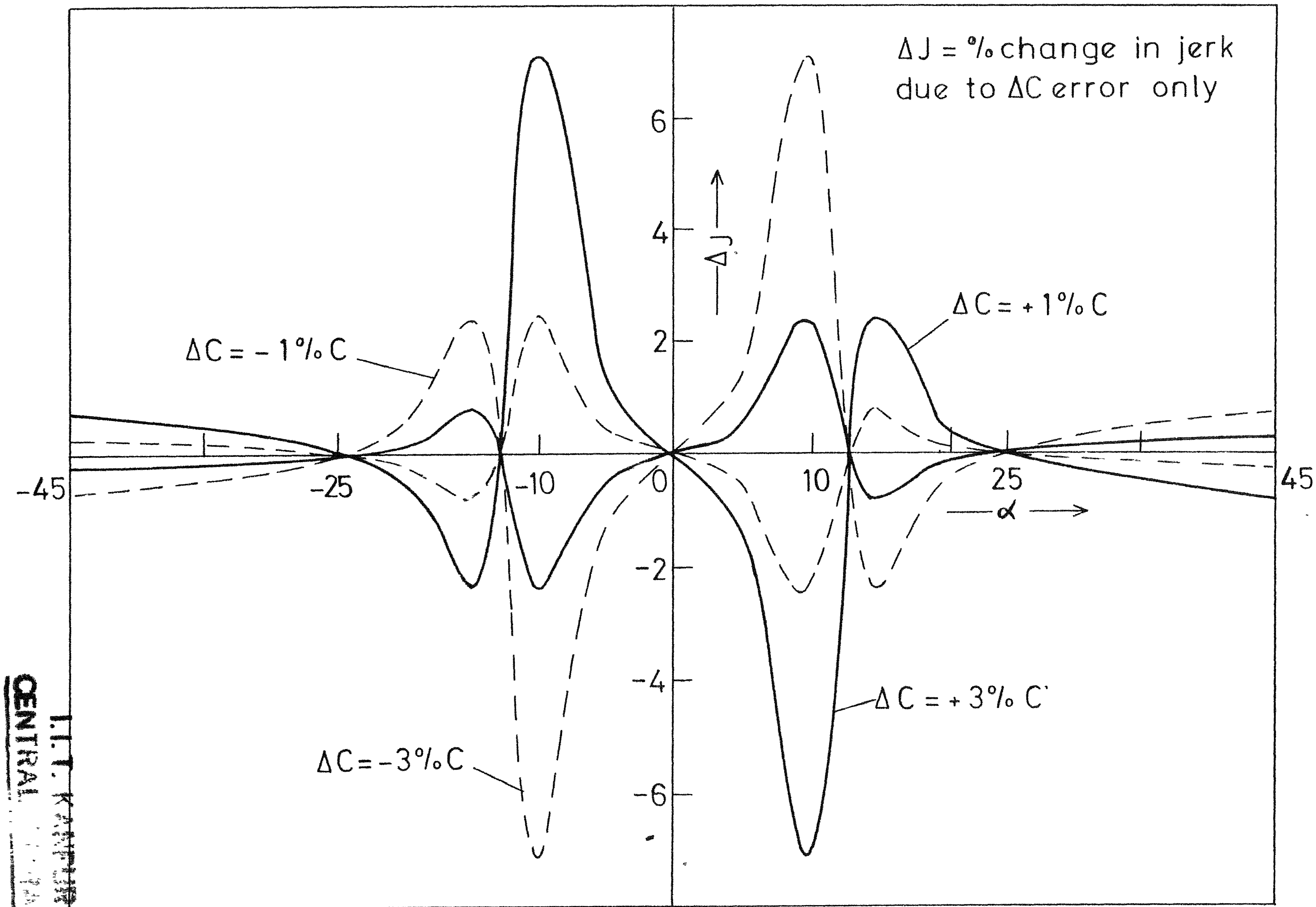


FIG. 2.8

DETERMINISTIC ERROR ANALYSIS OF EXTERNAL GENEVA MECHANISM

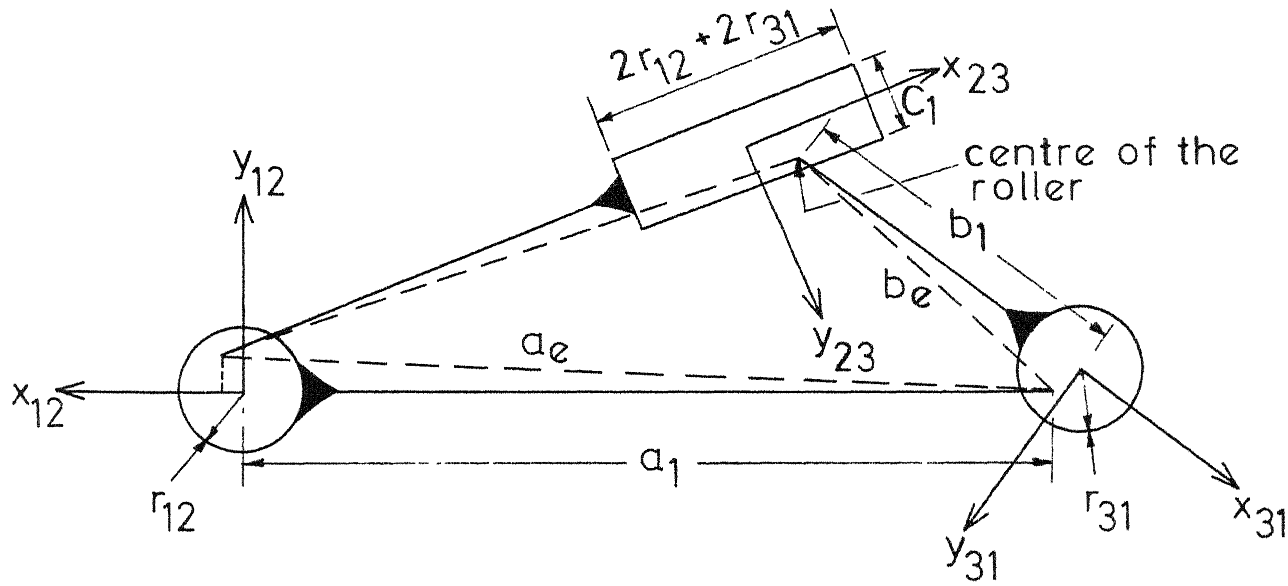


FIG. 2.9 PROBABILISTIC ERROR ANALYSIS CONSIDERING TOLERANCES AND CLEARANCES

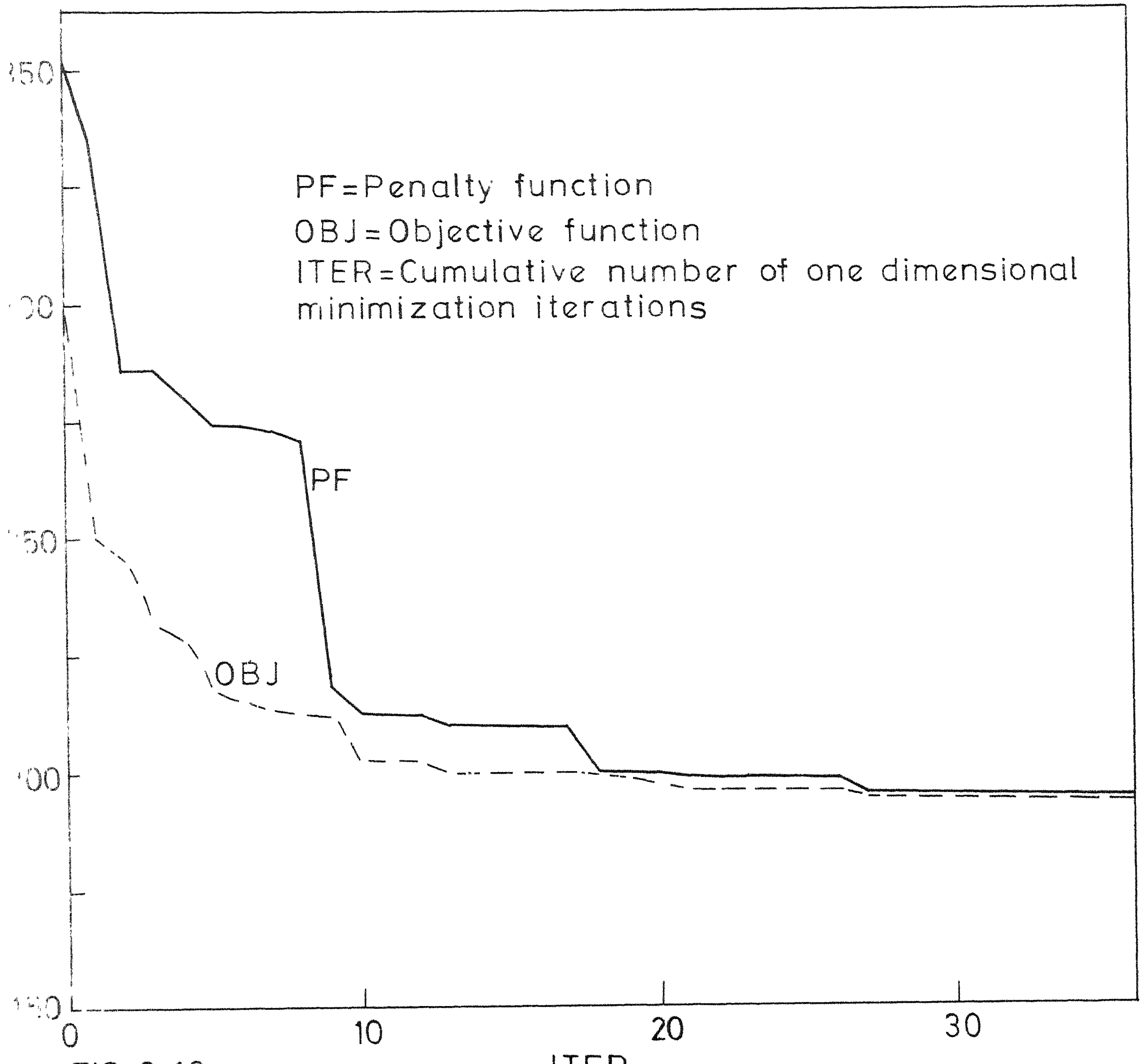


FIG. 2·10
PROGRESS OF OPTIMIZATION FOR EXTERNAL GENEVA MECHANISM

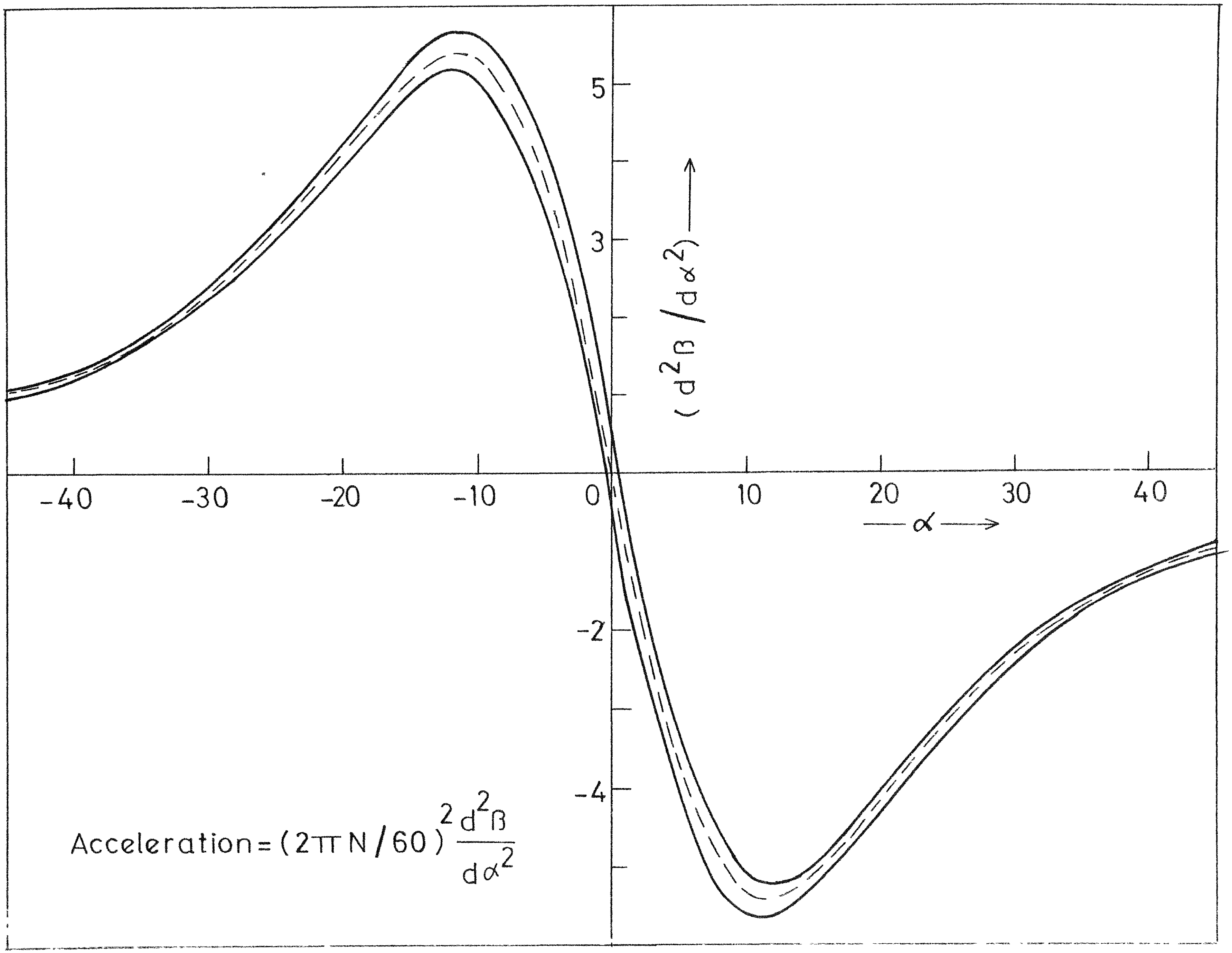

5
3
-2
-4
$(d^2\beta / d\alpha^2) \longrightarrow$
-40
-30
-20
-10
0
10
20
30
40
$\alpha \longrightarrow$
Acceleration $= (2\pi N/60)^2 \dfrac{d^2\beta}{d\alpha^2}$

FIG. 2.11

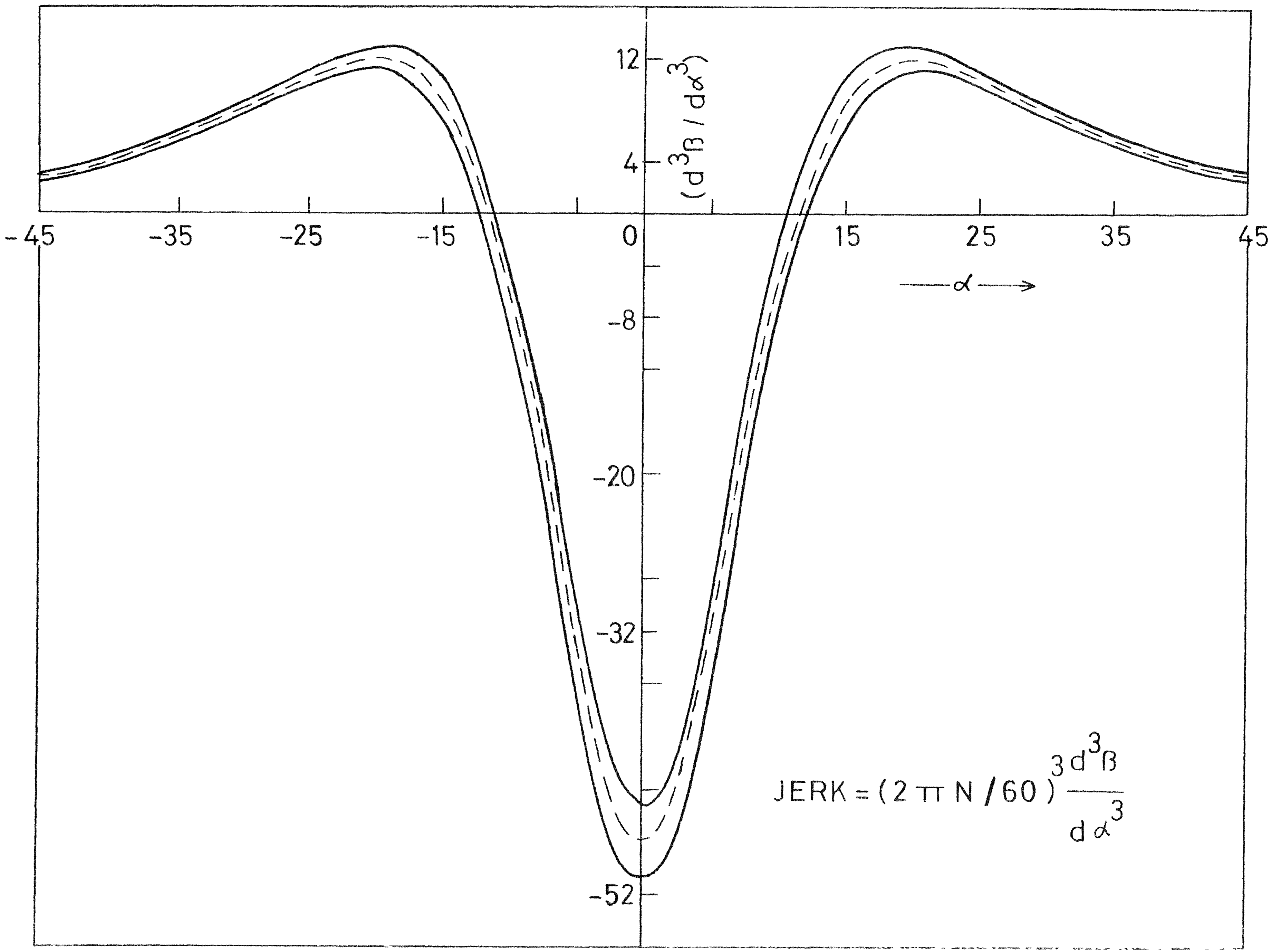


FIG. 2.12

[illegible] BAND FOR JERK AT THE [illegible] POINT [illegible]

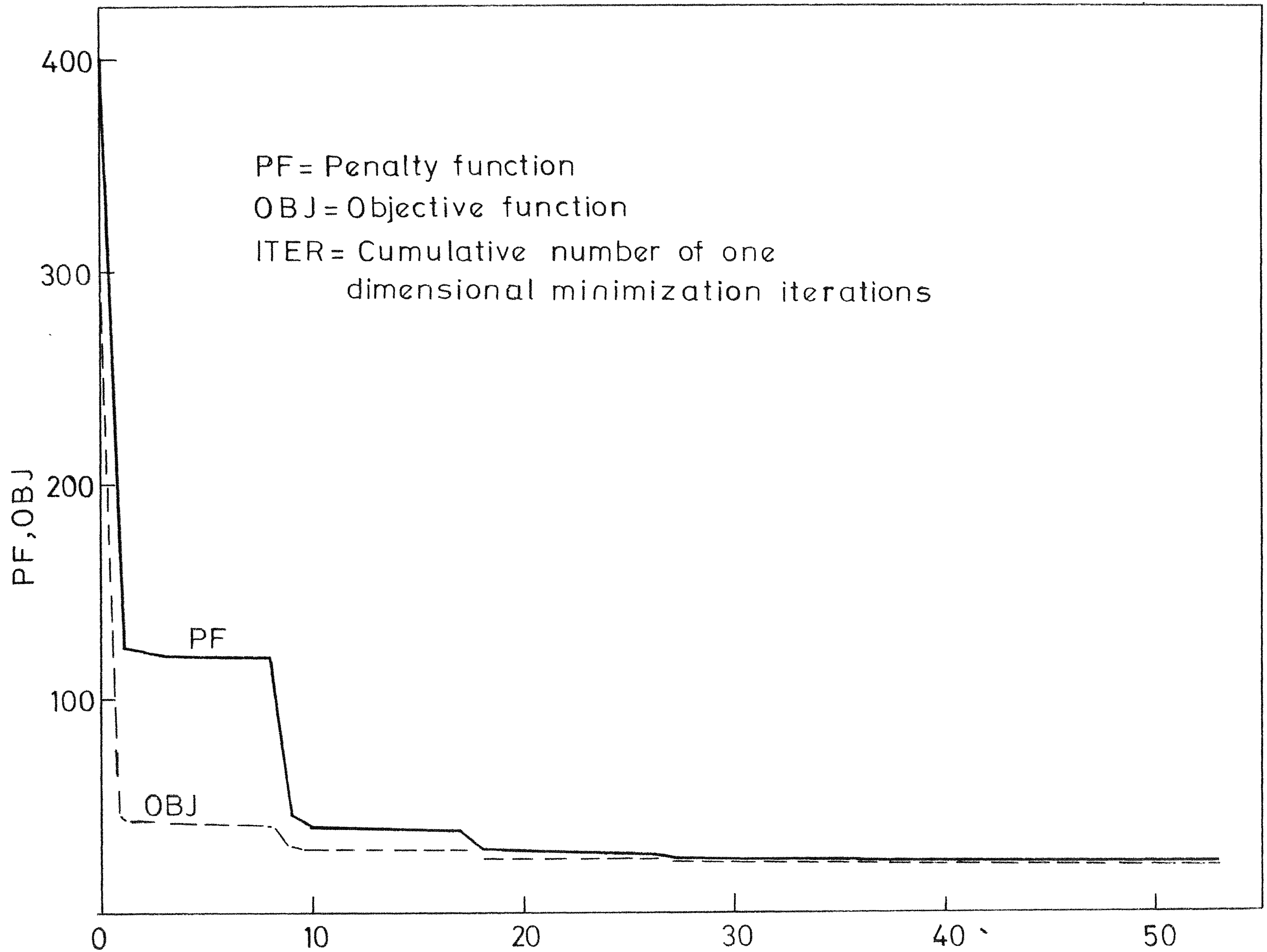


FIG. 2·13

PROGRESS OF OPTIMIZATION FOR INTERNAL GENEVA MECHA-

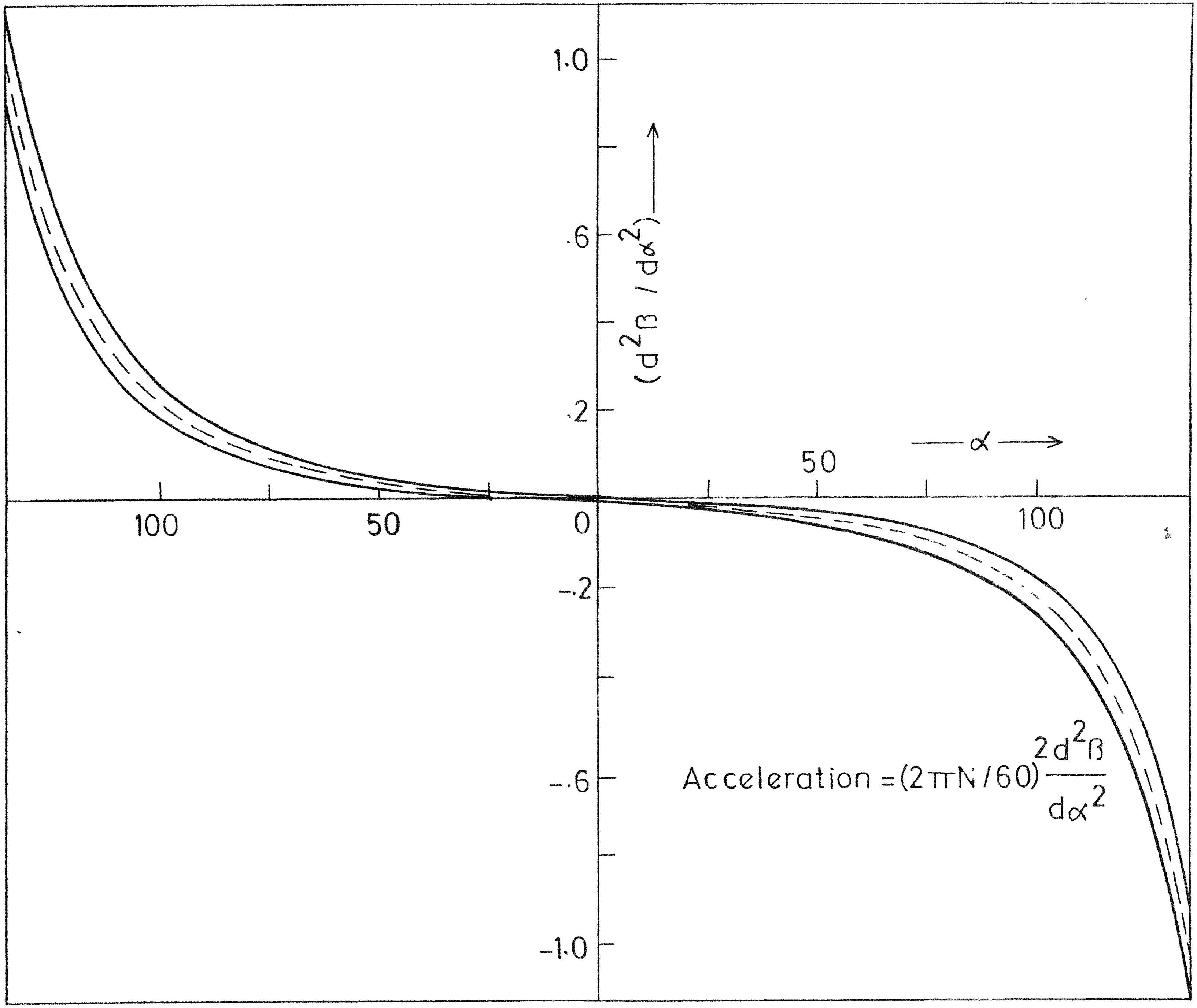


FIG. 2.14
3 σ BAND FOR ACCELERATION AT THE OPTIMUM POINT FOR
...TERNAL GENEVA MECHANISM

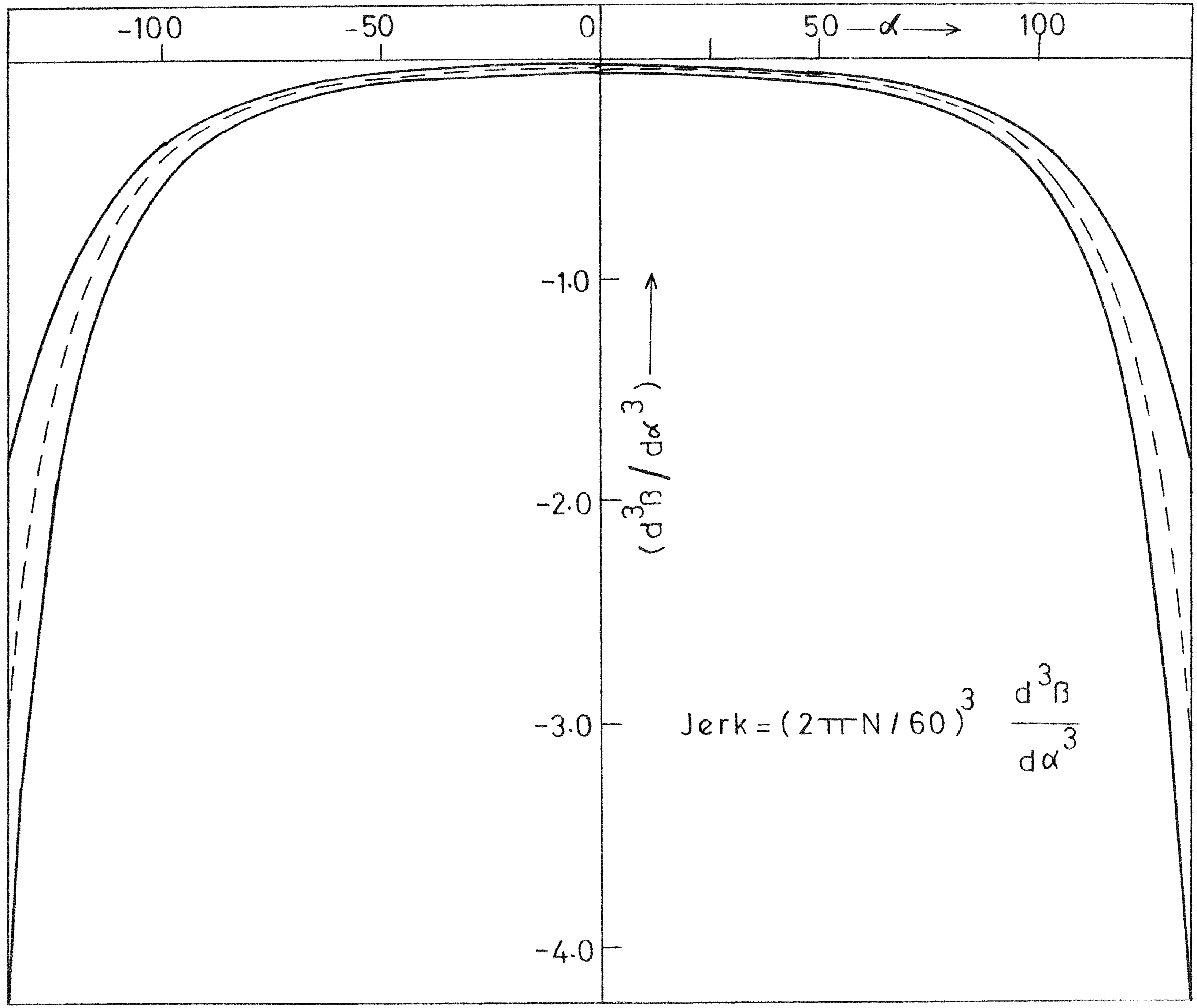


FIG. 2.15

3σ BAND FOR JERK AT THE OPTIMUM POINT FOR INTERNAL

CHAPTER 3

CAM - FOLLOWER SYSTEM

3.1 GENERAL DESCRIPTION:

In this chapter, the follower response of cam-follower systems is calculated probabilistically by considering the errors in various design parameters for the following two cases:

1. Effect of errors in the geometrical parameters on the kinematic response.
2. Effect of errors in geometrical and other system parameters (such as equivalent mass, stiffness and damping) on the dynamic response.

For simplicity, the analysis is carried for a two dimensional disc cam with translating roller follower. The kinematic response has been calculated for rise period only while the dynamic response has been computed for both rise and dwell periods.

The synthesis problems, involving the allocation of tolerances on the system parameters have also been solved using nonlinear programming techniques.

3.2 KINEMATIC RESPONSE OF THE FOLLOWER:

Figure 3.1 shows a disc cam with translating roller follower. The nomenclature is given below:

r = roller radius (cm)

r_b = radius of the base circle of the cam (cm.)

a = offset or eccentricity (cm.)

F_1 = coordinatesystem fixed to the follower (X_1, Y_1)

F_2 = coordinate system fixed to the cam disc (X_2, Y_2)

ϕ_2 = cam rotation angle

θ = parameter of the follower surface

w_2 = constant angular velocity of the cam shaft (radians/secs.)

s_1 = displacement of the follower (cm.)

L_o = total lift of the follower (cm.)

$\vec{X}$ = design vector $(x_1, x_2, x_3, x_4, x_5)^T$ corresponding to the tolerances in the geometrical parameters $(\Delta r, \Delta r_b, \Delta a, \Delta X, \Delta Y)^T$

ϕ_o = cam angle for total rise

$s'_1 = \dfrac{d\, s_1}{d\, \phi_2}$

$$s_o = (r_b^2 - a^2)^{1/2} \qquad (3.1)$$

X_2, Y_2 = coordinates of the point of contact P.

The condition of contact can be derived as follows [9] :

$$E \sin \theta + F \cos \theta + G = 0 \qquad (3.2)$$

where

$E = s'_1 - a$,

$F = s_1 + s_o$,

and $G = 0$.

From this equation, the parameter θ can be readily evaluated as

$$\theta = \tan^{-1} \left(\frac{s_1 + s_o}{s_1' - a} \right) \tag{3.3}$$

The coordinates of the cam profile can be derived as |9| :

$$X_2 = - r \operatorname{Cos} (\theta + \phi_2) + a \operatorname{Cos} \phi_2 + (s_1 + s_o) \operatorname{Sin} \phi_2 \tag{3.4}$$

$$Y_2 = - r \operatorname{Sin} (\theta + \phi_2) - a \operatorname{Sin} \phi_2 + (s_1 + s_o) \operatorname{Cos} \phi_2 \tag{3.5}$$

Using these equations, the derived relation for follower response for error analysis can be derived as

$$s_1 = X_2 \operatorname{Sin} \phi_2 + Y_2 \operatorname{Cos} \phi_2 - r \sin \theta - s_o \tag{3.6}$$

Equation (3.6) indicates that s_1, the follower response, is not an explicit function of the geometrical parameters, because the variables θ, X_2 and Y_2 themselves are functions of s_1 as given by Eqs. (3.3) to (3.5), respectively.

The tolerances in the geometrical lengths 'r' and 'r_b' are, obviously, random variables. Further some error is bound to occur in fixing the cam follower system and hence the **offset** or eccentricity 'a' can be assumed as a random variable. In manufacturing the cam, errors are unavoidable. Thus, the tolerances in the coordinates of

the cam profile can be assumed as random variables. Though, these coordinates change with the cam rotation angle, the tolerance ΔX in the X_2 coordinate and ΔY in the Y_2 coordinate are supposed to be the same for all the cam rotation angles. It is assumed here that the workmanship of the manufacturer remains the same.

To start with, a theoretical s_1 is assumed and substituted in Eqs. (3.3) to (3.5) to get θ, X_2 and Y_2 respectively. These values are then substituted in Eq. (3.6) to get s_1. The standard deviation of the kinematic response s_1 can be obtained by applying the partial derivative rule as

$$(\sigma_{s_1})_j = \left[\sum_{i=1}^{5} \left(\frac{\partial s_{1j}}{\partial x_i} \right)^2 \Big|_{\vec{\underline{X}}} \sigma_{x_i}^2 \right]^{1/2} \qquad (3.7)$$

where

$(\sigma_{s_1})_j$ = standard deviation of s_1 at $\phi_2 = (\phi_2)_j$

$(s_1)_j$ = s_1 at $\phi_2 = (\phi_2)_j$

$\vec{\underline{X}}$ = mean design vector

= $(0.0, 0.0, 0.0, 0.0, 0.0)^T$

The partial derivatives, required for the computation of $(\sigma_{s_1})_j$ are given below:

$$\frac{\partial s_1}{\partial x_1} = \text{Sin } \phi_2$$

$$\frac{\partial s_1}{\partial x_2} = \text{Cos } \phi_2$$

$$\left.\begin{aligned}
\frac{\partial s_1}{\partial x_3} &= - \sin\theta \\
\frac{\partial s_1}{\partial x_4} &= - \Upsilon \cos\theta \cdot \frac{\partial \theta}{\partial r_b} - \frac{\partial s_o}{\partial r_b} \\
\frac{\partial s_1}{\partial x_5} &= - \Upsilon \cos\theta \frac{\partial \theta}{\partial a} - \frac{\partial s_o}{\partial a} \\
\frac{\partial \theta}{\partial r_b} &= \left(\frac{\partial s_o}{\partial r_b}\right)(s_1' - a) \Big/ \left[(s_1' - a)^2 + (s_1 + s_o)^2\right] \\
\frac{\partial \theta}{\partial a} &= \left(\frac{\partial s_o}{\partial a}\right)(s_1' - a) \Big/ \left[(s_1' - a)^2 + (s_1 + s_o)^2\right] \\
\frac{\partial s_o}{\partial r_b} &= r_b / (r_b^2 - a^2)^{1/2} \\
\frac{\partial s_o}{\partial a} &= - a / (r_b^2 - a^2)^{1/2}
\end{aligned}\right\} \quad (3.8)$$

PROCEDURE TO FIND THE MEAN AND STANDARD DEVIATION OF THE KINEMATIC RESPONSE

The various steps involved in the computation of the mean and the standard deviation of the follower response are given below:

1. The dam angle for total lift, that is $\emptyset_o$, is divided into a convenient finite number of intervals (n - 1). The discrete values of $\emptyset_2$ are numbered as $(\emptyset_2)_j$, $j = 1, 2, \ldots n$. Set $j = 1$.

2. A suitable lift curve is assumed. Here cycloidal motion has been selected which gives motion with much reduced discontinuity of acceleration compared with other common types of motions. The lift s_1,

corresponding to any cam angle ϕ_2, according to cycloidal motion is given by

$$s_1 = \frac{L_o}{\pi}\left[\pi \phi_2/\phi_o - (\tfrac{1}{2}) \operatorname{Sin}(2\pi \phi_2/\phi_o)\right] \qquad (3.9)$$

and hence

$$s_1' = \frac{d\, s_1}{d\, \phi_2} = \left(\frac{L_o}{\phi_o}\right)\left[1 - \operatorname{Cos}(2\pi \phi_2 / \phi_o)\right] \qquad (3.10)$$

The values of s_1 and s_1' are calculated for a particular value of $\phi_2 = (\phi_2)_j$ and are then substituted in Eq. (3.3) to get the corresponding value of θ at $\phi_2 = \phi_{2j}$ (that is θ_j).

3. The mean lift $(s_1)_j$ and standard deviation $(\sigma_{s_1})_j$ are calculated using Eqs. (3.6) and (3.7) respectively.
4. Set new j = present $j + 1$ and repeat steps 2 through 5 until the angle $(\phi_2)_n$ is exhausted.

3.3 OPTIMUM ALLOCATION OF TOLERANCES

In most of the mechanisms that use cam follower systems, the follower response is not allowed to deviate more than a prescribed value from the mean (or nominal) position. The motto of the optimization problem is to allocate maximum tolerances to the various geometrical parameters (and hence minimize the manufacturing cost) keeping the follower response within the tolerable limits.

The tolerances for all the geometrical parameters are assumed to be normally distributed with $\pm 3\sigma$ band. Hence, if the mean value of one of the parameters is r and its standard deviation is σ_r, then the allowable value will be $(r \pm 3\sigma_r)$. Hence $\sigma_r = \Delta r/3 = x_3/3$ where Δr is the tolerance given to r.

Similarly

$$\sigma_{rb} = \Delta r_b/3 = x_4/3$$
$$\sigma_a = \Delta a/3 = x_5/3$$
$$\sigma_{X_2} = \Delta X/3 = x_1/3$$
$$\sigma_{Y_2} = \Delta Y/3 = x_2/3$$

The optimization problem is formulated as follows:

Find $\vec{X} = (x_1, x_2, x_3, x_4, x_5)^T$

which minimizes

$$OBJ = \sum_{i=1}^{5} \frac{1}{x_i}$$

Subject to the following constraints

$$l_i \leq x_i \leq u_i \quad , \qquad i = 1 \text{ to } 5$$
$$(\sigma_{s_1})_j \leq E$$

where

l_i = lower bound on x_i,

u_i = upper bound on x_i, and

E = upper bound on the standard deviation of the follower response.

NUMERICAL EXAMPLE

For solving the optimization problem the following data has been utilised.

$$\{l_i\} = (0.001, \quad 0.001, \quad 0.3\%\ r, \quad 0.3\%\ r_b, \quad 0.3\%\ a)$$

$$\{u_i\} = (0.2, \quad 0.2 \quad , \quad 3.0\%\ r, \quad 3.0\%\ r_b, \quad 3.0\%\ a)$$

$$r = 1.0, \qquad r_b = 2.5, \qquad a = 1.2$$

Total angle of rise $\phi_o = 150^\circ$.

The problem has been solved for four different values of E, namely, .01, .06, 0.10 and 0.20.

The results of optimization are summarized in Tables 3.1 to 3.4. It can be seen that the objective function has been reduced by 70.5% for $E = 0.01$, 90.7% for $E = 0.06$, 91.5% for $E = 0.1$ and 91.8% for $E = 0.2$. The progress of optimization for $E = 0.1$ is shown in Fig. 3.2. The objective function decreases rapidly in the first few iterations for all E's except for $E = 0.01$. The standard deviation of the response corresponding to various values of ϕ_2 at the optimum point are given in Table 3.5 for different values of E. It is important to note that the standard deviation almost remains the same for all values of ϕ_2 at any particular $\overset{*}{X}$. This is the reason why at the optimum point corresponding to $E = 0.01$, all the upper bounds on the standard deviation of lift are critical.

For E = .06, E = 0.1 and E = 0.2, none of the constraints corresponding to the upper bounds on the standard deviation of lift is critical, revealing that the upper bound has been relaxed too much. There is no significant improvement in the objective function at the optimum point by relaxing the upper bound on the standard deviation of lift from E = .06 to E = 0.2. In most of the practical cam follower systems, the value of E is to be restricted to be less than or equal to 0.01.

3.4 DYNAMIC RESPONSE OF THE CAM FOLLOWER SYSTEM

The kinematic analysis alone will not be sufficient for high speed cam mechanisms. It needs to be supplemented by the dynamic analysis to ensure the faithful reproduction of the follower motion. As the exact mathematical treatment is extremely complicated, mathematical modelling of the whole mechanism is necessary. Assuming the cam shaft to be absolutely rigid, the system can be idealized as a two degree of freedom system {19}. By reducing the load and the entire follower train into a single mass m, the system can be further simplified so that it has a single degree of freedom as shown in Figure 3.3.b.

The objective of the dynamic analysis of the cam system is to determine the response x when s_1 is known as a function of time.

The nomenclature used is given below:

M	=	Equivalent mass of the follower train (Kg)
k_1	=	Stiffness of the follower spring (Kg/cm.)
k_2	=	Stiffness of the follower train (Kg/cm.)
c_1	=	Damping coefficient of follower (Kg-sec./cm.)
c_2	=	Damping coefficient of follower train (Kg-sec./cm)
N	=	Speed of the cam shaft (rpm)
ω	=	Angular velocity of cam shaft = $\frac{2\pi N}{60}$ radians/sec.
s_1	=	Kinematic response of the cam (cm)
$\overset{o}{s}_1$	=	$\frac{d\, s_1}{dt}$
x	=	Dynamic response of the cam follower system (cm.)
$\overrightarrow{XX}$	=	$(r, r_b, a, m, k_1, k_2, c_1, c_2, X_2, Y_2)$
$(\sigma_{x_i})_r$	=	Standard deviation of lift during the rise period at the ith time step
$(\sigma_{x_i})_d$	=	Standard deviation of lift during the dwell period at the ith time step
$\vec{X}$	=	Design vector = $(\Delta r, \Delta r_b, \Delta a, \Delta m, \Delta k_1, \Delta k_2, \Delta c_1, \Delta c_2, \Delta X, \Delta Y)$
$\vec{U}$	=	Vector of upper bounds on the variables $\vec{X}$
$\vec{L}$	=	Vector of lower bounds on the variables $\vec{X}$
E	=	Upper bound on the standard deviation of lift (cm)

Numerical Solution for Dynamic Response During the Rise Period

The dynamic equilibrium equation of the system shown in Fig. 3.3.b is given by

$$m\ddot{x} + (c_1 + c_2)\dot{x} + (k_1 + k_2)x = k_2 s_1 + c_2 \dot{s}_1$$

or $$\ddot{x} + \left(\frac{c_1 + c_2}{m}\right)\dot{x} + \left(\frac{k_1 + k_2}{m}\right)x = \left(\frac{k_2 s_1 + c_2 \dot{s}_1}{m}\right) \quad (3.11)$$

The right hand side of the above second order linear differential equation, with constant coefficients, is time dependent. Here the kinematic response (s_1) is given by the following expression

$$s_1 = X_2 \operatorname{Sin} \phi_2 + Y_2 \operatorname{Cos} \phi_2 - r \operatorname{Sin} \theta - s_o \quad (3.12)$$

It is not possible to get the time derivative of s_1 explicitly (as the time derivatives of X_2 and Y_2 are not known) and hence the dynamic response x can not be obtained as a closed form expression. In this work $\dot{s}_1$ is calculated by numerical differentiation.

Equation (3.11) can be written as

$$\ddot{x} + 2n\dot{x} + p^2 x = q_i \quad (3.13)$$

where $n = \dfrac{c_1 + c_2}{2m}$,

$p^2 = \dfrac{k_1 + k_2}{m}$,

and $q_i = \dfrac{Q_i}{m}$ = forcing function per unit mass.

In most practical cam mechanisms the values of n and p are such that the roots of the auxiliary equation

$\ddot{x} + 2 n \dot{x} + p^2 = 0$ are complex and hence the system under consideration is underdamped. The transient response is given by

$$x = e^{-nt} (A_1 \text{Cos } p_d t + A_2 \text{Sin } p_d t) \qquad (3.14)$$

where

$$p_d = \left[p^2 - n^2 \right]^{1/2}$$

To determine the constants A_1 and A_2, it is assumed that at the initial instant ($t = 0$), the vibrating body is displaced from its position of equilibrium by the amount x_o and has an initial velocity $\dot{x}_o$. Hence

$$A_1 = x_o \quad \text{and} \quad A_2 = \frac{\dot{x}_o + n x_o}{p_d}$$

$$\text{Hence transient response} = e^{-nt} \left(x_o \cos p_d t + \frac{\dot{x}_o + n x_o}{p_d} \text{Sin } p_d t \right).$$

The response for an arbitrary disturbing force $q = f(t')$ is given by the Duhamel's integral as [15]:

$$x = \frac{e^{-nt}}{p_d} \int_0^t e^{nt'} \cdot q \cdot \text{Sin } p_d (t - t')\, dt'$$

The complete solution is given by the following expression:

$$x = e^{-nt}\left[x_o \operatorname{Cos} p_d t + \frac{\dot{x}_o + n x_o}{p_d} \operatorname{Sin} p_d t + \frac{Q_1}{p_d} \int_o^t e^{nt'} q \operatorname{Sin} p_d (t - t')\, dt'\right] \tag{3.15}$$

As a particular case when the forcing function is a step function, as shown in Fig. 3.4, the response is given by

$$x = e^{-nt}\left[x_o \operatorname{Cos} p_d t + \frac{\dot{x}_o + n x_o}{p_d} \operatorname{Sin} p_d t + \frac{q_1}{p_d} \int_o^t e^{nt'} \operatorname{Sin} p_d (t - t')\, dt'\right]$$

where $q = \frac{Q_1}{m}$.

The integral in the above equation is integrated by parts to get

$$x = e^{-nt}\left[x_o \operatorname{Cos} p_d t + \frac{\dot{x}_o + n x_o}{p_d} \operatorname{Sin} p_d t\right] + \frac{Q_1}{m}\left[1 - e^{-nt}\left(\operatorname{Cos} p_d t + \frac{n}{p_d} \operatorname{Sin} p_d t\right)\right]$$

The usual procedure is to approximate the forcing function by a series of step functions as shown in Fig. 3.4. Care is taken to see that the time step is sufficiently small compared to the natural time period of the system. In any time interval $t_{i-1} \leqslant t \leqslant t_i$, the response of the one degree system may be calculated as the sum of the effects of the initial conditions at time t_{i-1} and the effect of the impulse within the interval Δt_i, as follows:

$$x = e^{-n(t - t_{i-1})} \left[x_{i-1} \cos p_d (t - t_{i-1}) + \frac{\dot{x}_{i-1} + n x_{i-1}}{p_d} \sin p_d (t - t_{i-1}) \right] + \frac{q_i}{p^2} \left\{ 1 - e^{-n(t - t_{i-1})} \left[\cos p_d (t - t_{i-1}) + \frac{n}{p_d} \sin p_d (t - t_{i-1}) \right] \right\}$$

At the end of the interval this expression becomes

$$x_i = e^{-n \Delta t_i} \left[x_{i-1} \cos p_d \Delta t_i + \frac{\dot{x}_{i-1} + n x_{i-1}}{p_d} \sin p_d \Delta t_i \right] + \frac{q_i}{p^2} \left[1 - e^{-n \Delta t_i} \left(\cos p_d \Delta t_i + \frac{n}{p_d} \sin p_d \Delta t_i \right) \right] \tag{3.16}$$

The velocity can be determined from the following expre-ssion

$$\frac{\dot{x}_i}{p_d} = e^{-n \Delta t_i} \left[- x_{i-1} \sin p_d \Delta t_i + \frac{\dot{x}_{i-1} + n x_{i-1}}{p_d} \cos p_d \Delta t_i - \frac{n}{p_d} \left(x_{i-1} \cos p_d \Delta t_i + \frac{\dot{x}_{i-1} + n x_{i-1}}{p_d} \sin p_d \Delta t_i \right) \right] + \frac{q_i}{p^2} \cdot e^{-n \Delta t_i} \left(1 + \frac{n^2}{p_d^2} \right) \sin p_d \Delta t_i \tag{3.17}$$

Equations (3.16) and (3.17) represent the recurrence formulas for calculating the dynamic response at the end of the ith time step. They also provide the

initial conditions of displacement and velocity at the beginning of $(i + 1)^{th}$ step. Continuing this way, the displacement $x(\phi_o)$ and velocity $\dot{x}(\phi_o)$ are found.

Analytical Solution for Dynamic Response During the Dwell Period

During the dwell period the rise of the cam is constant and is equal to L_o. Substituting $s_1 = L_o$ and $\dot{s}_1 = 0$ in Eq. (3.11) the dynamic equilibrium equation modifies to

$$\ddot{x} + \left(\frac{c_1 + c_2}{m}\right)\dot{x} + \left(\frac{k_1 + k_2}{m}\right)x = \frac{k_2 L_o}{m} \tag{3.18}$$

The solution of this second order linear differential equation with constant coefficient gives

$$x = e^{-nt}(A_3 \operatorname{Cos} p_d t + A_4 \operatorname{Sin} p_d t) + \frac{k_2 L_o}{k_1 + k_2}$$

The initial conditions used to calculate the constants A_3 and A_4, are as follows:

$$\text{At} \quad t = \frac{\phi_o \pi}{180\,\omega}, \quad x = x(\phi_o)$$

$$\text{At} \quad t = \frac{\phi_o \pi}{180\,\omega} \quad \dot{x} = \dot{x}(\phi_o)$$

The constants can be evaluated as

$$A_3 = x(\phi_o) - \frac{k_2 L_o}{k_1 + k_2}$$

$$A_4 = (\dot{x}(\phi_o) + n A_3) / p_d$$

Calculation of Standard Deviation of Response

The partial derivatives required for the calculation of the standard deviation of lift during the rise period are calculated by numerical differentiation. The standard deviation is given by

$$(\sigma_{x_i})_r = \left[\sum_{k=1}^{10} \left(\frac{\partial x_i}{\partial XX(k)} \right)^2 \sigma^2_{XX(k)} \right]^{1/2} \qquad (3.19)$$

The partial derivatives needed for the calculation of standard deviation of lift during the dwell period are calculated analytically. It can be seen from equation (3.18) that the dynamic response in the dwell period is independent of the geometrical parameters. Hence

$$(\sigma_{x_i})_d = \left[\sum_{k=4}^{8} \left(\frac{\partial x_i}{\partial XX(k)} \right)^2 \sigma^2_{XX(k)} \right]^{1/2} \qquad (3.20)$$

where

$$\frac{\partial x_i}{\partial XX(k)} = e^{-nt} \left(- A_3 \sin p_d t \cdot p_d \frac{\partial p_d}{\partial XX(k)} + A_4 \cos p_d t \cdot pd \frac{\partial p_d}{\partial XX(k)} \right) + \frac{\partial (CON)}{\partial XX(k)}$$

$$- e^{-nt} n \frac{\partial n}{\partial XX(k)} (A_3 \cos p_d t + A_4 \sin p_d t),$$

$$CON = \frac{k_2 L_o}{k_1 + k_2},$$

$$\left[\begin{array}{l} \frac{\partial n}{\partial XX(4)} = - \frac{n}{m}, \quad \frac{\partial n}{\partial XX(5)} = \frac{\partial n}{\partial XX(6)} = 0 \\ \frac{\partial n}{\partial XX(7)} = \frac{\partial n}{\partial XX(8)} = \frac{1}{2m} \end{array} \right.$$

Calculation of Standard Deviation of Response

The partial derivatives required for the calculation of the standard deviation of lift during the rise period are calculated by numerical differentiation. The standard deviation is given by

$$(\sigma_{x_i})_r = \left[\sum_{k=1}^{10} \left(\frac{\partial x_i}{\partial XX(k)} \right)^2 \sigma^2_{XX(k)} \right]^{1/2} \tag{3.19}$$

The partial derivatives needed for the calculation of standard deviation of lift during the dwell period are calculated analytically. It can be seen from equation (3.18) that the dynamic response in the dwell period is independent of the geometrical parameters. Hence

$$(\sigma_{x_i})_d = \left[\sum_{k=4}^{8} \left(\frac{\partial x_i}{\partial XX(k)} \right)^2 \sigma^2_{XX(k)} \right]^{1/2} \tag{3.20}$$

where

$$\frac{\partial x_i}{\partial XX(k)} = e^{-nt} \left(- A_3 \operatorname{Sin} p_d t \cdot p_d \frac{\partial p_d}{\partial XX(k)} + A_4 \operatorname{Cos} p_d t \cdot pd \frac{\partial p_d}{\partial XX(k)} \right) + \frac{\partial (CON)}{\partial XX(k)} - e^{-nt} n \frac{\partial n}{\partial XX(k)} (A_3 \operatorname{Cos} p_d t + A_4 \operatorname{Sin} p_d t),$$

$$CON = \frac{k_2 L_o}{k_1 + k_2},$$

$$\left[\begin{array}{l} \dfrac{\partial n}{\partial XX(4)} = -\dfrac{n}{m}, \quad \dfrac{\partial n}{\partial XX(5)} = \dfrac{\partial n}{\partial XX(6)} = 0 \\ \dfrac{\partial n}{\partial XX(7)} = \dfrac{\partial n}{\partial XX(8)} = \dfrac{1}{2m} \end{array} \right.$$

$$\frac{\partial p_d}{\partial XX(4)} = -\frac{p_d}{m}, \quad \frac{\partial p_d}{\partial XX(5)} = \frac{\partial p_d}{\partial XX(6)} = \frac{p}{m \cdot p_d}$$

$$\frac{\partial p_d}{\partial XX(7)} = \frac{\partial p_d}{\partial XX(8)} = -\frac{n}{2\, m\, p_d}$$

$$\frac{\partial CON}{\partial XX(4)} = \frac{\partial CON}{\partial XX(7)} = \frac{\partial CON}{\partial XX(8)} = 0$$

$$\frac{\partial CON}{\partial XX(5)} = \frac{-CON}{k_1 + k_2}, \quad \frac{\partial CON}{\partial XX(6)} = \frac{-CON}{k_1 + k_2} + \frac{L_o}{k_1 + k_2}$$

3.5 ALLOCATION OF OPTIMUM TOLERANCES:

The tolerances in the geometrical parameters and the variations in the system parameters are assumed to be normally distributed. The optimization problem is formulated as follows:

$$\text{Find } \vec{X} = (\Delta r, \Delta r_o, \Delta a, \Delta m, \Delta k_1, \Delta k_2, \Delta c_1, \Delta c_2, \Delta X, \Delta Y)$$

which minimizes

$$OBJ = \sum_{i=1}^{10} \frac{1}{X(I)}$$

subject to the following constraints

$$L(I) \leq X(I) \leq U(I), \quad I = 1, 10$$

$$(\sigma x_i) \leq E, \quad i = 1, 2, \ldots n.$$

where $L(I)$ = Lower bound on $X(I)$ random variable

$U(I)$ = Upper bound on $X(I)$ random variable

(σx_i) = Standard deviation at ith step

E = Upper bound on the standard deviation of the dynamic response x.

NUMERICAL EXAMPLE

For solving the optimization problem the following data has been utilised.

$$U(I) = 3\%\ XX(I) \quad , \quad I = 1, 3$$
$$U(I) = 10\%\ XX(I) \quad , \quad I = 4, 8$$
$$U(9) = U(10) = 0.2$$
$$L(I) = 0.2\%\ XX(I) \quad , \quad I = 1, 3$$
$$L(I) = 2\%\ XX\ (I) \quad , \quad I = 4, 8$$
$$L(9) = L(10) = 0.001$$

Rise period = 0 to 150°

Dwell period = 150° to 210°

$$XX(I) = (r, r_b, a, m, k_1, k_2, c_1, c_2)$$
$$= (1.0, 2.5, 1.2, 0.3, 40.0, 700.0, 0.005, 0.005)$$

$$N = 3000 \text{ r.p.m.}$$

Figure 3.5 shows the mean dynamic response, the assumed motion and the follower command for various values of ϕ_2.

The optimization problem has been solved for three different values of E, namely, 0.06, 0.1 and 0.2.

The results of optimization are summarized in Tables 3.6 to 3.8. It can be seen that the objective function has been reduced by 26.1% for E = 0.06, 57.0% for E = 0.1 and 66.8% for E = 0.2. Once again the objective function decreases rapidly in the first few iterations. The progress of optimization for all E's is shown in Fig. 3.6. The standard deviations corresponding to the various values of ϕ_2 at the optimum point for different values of E are given in Table 3.9. It is to be noted that the standard deviation increases with the increase in the dynamic response upto the rise period. From Tables 3.6 to 3.8 it can be observed that as the value of E decreases from 0.2 to .06, more and more constraints corresponding to the upper bound on the standard deviation of lift become critical. From Table 3.9, it is possible to plot $\pm 3\sigma$ band about the mean dynamic response for any value of E.

It can be concluded by observing the standard deviation of the kinematic response given in Table 3.5 and the standard deviation of the dynamic response given in Table 3.9, that the former is much smaller compared to the later. Thus the consideration of kinematic analysis alone is not sufficient to synthesize the system. It has to be supplemented by the dynamic analysis which gives more realistic results.

TABLE 3.1

Starting feasible point $\vec{X}_1$ = (0.01, 0.01, 0.003, 0.006, 0.003)
Initial penalty parameter r_1 = 1.0

Initial objective function OBJ = 1033.333
Initial penalty function PF = 1108.27
Upper bound on the standard deviation of kinematic response E = 0.01
Number of iterations for minimizing PF = NITER

K	R_k	NITER	OPTIMUM POINT $\vec{X}_k$	PF	OBJ
1	1.0×10^{0}	10	(0.013188, 0.0144400, 0.011199, 0.010064,0.016205)	535.81	395.43
2	1.0×10^{-1}	10	(0.0158608, 0.0172831, 0.013432, 0.011983, 0.0195113)	364.69	330.05
3	1.0×10^{-2}	10	(0.017092, 0.018140, 0.014291, 0.012762, 0.0209954)	319.810	309.59
4	1.0×10^{-3}	10	(0.0181938, 0.019004, 0.0136145, 0.0131815, 0.020763)	307.8	305.06
5	1.0×10^{-4}	3	(0.0182279, 0.0189505, 0.0136106, 0.0133780,0.0207960)	304.6	303.93

Active constraints are as follows: 66(11) to 66(41).

TABLE 3.2

Starting feasible point $\vec{X_1}$ = (.01,.01, .003, .006, .003)
Initial value of the penalty function at $\bar{X_1}$ = 1088.275
Objective function at the starting point $\vec{X_1}$ = 1033.333
Initial value of the penalty parameter r_1 = 1.0
Upper bound on the standard deviation of kinematic response E = 0.06

Number of iterations for minimizing PF = NITER

K	R_k	NITER	Optimum point $\vec{X_k}$	PF	OBJ
1	1.0×10^{0}	10	(.063702, 0.0663417, .0253191, .0453005, .0300726)	209.506	125.595
2	1.0×10^{-1}	10	(.100797, .105859, .028368, .0651468, .0338485)	116.160	99.511
3	1.0×10^{-2}	6	(.105070, .108697, .0297501, .0745678, .0349204)	99.161	94.377
4	1.0×10^{-3}	10	(.105070, .108782, .029861, .0746509, .0350324)	94.658	94.140
5	1.0×10^{-4}	5	(.105194, 0.108909, 0.0299730, .0749176, .035325)	93.920	93.708

GG'(3) = -.0009002835 = Upper bound on Δr
GG (4) = -.001098200 = Upper bound on Δr_1
GG'(5) = -.03837496 = Upper bound on Δa

TABLE 3.3

Starting feasible point $\vec{X_1}$ = (.01, .01, .003, .006, .003)

r_1 = 1.0

PF = 1087.304

OBJ = 1033.333

Upper bound on the standard deviation of kinematic response E = 0.1

NITER = Number of iterations for minimising PF

K	R_k	NITER	Optimum point $\vec{X_k}$	PF	OBJ
1	1.0×10^{0}	10	(.0875371, .0909304, .0255043, .0543696, .032205)	188.135	113.113
2	1.0×10^{-1}	10	(.157730, .158606, .0284038, .0685778, .0339118)	105.887	91.921
3	1.0×10^{-2}	7	(.165415, .164309, .029616, .0730761, .0346682)	90.863	88.426
4	1.0×10^{-3}	8	(.165423, .164317, .0298753*, .073107*, .0348470*)	88.389	87.978

Active Constraints

GG (3) = −.004157014 = Upper bound on Δr

GG (4) = −.02523983 = Upper bound on Δr_b

GG (5) = −.05138636 = Upper bound on $\Delta \alpha$

TABLE 3.4

Starting feasible point $\vec{X_1}$ = (0.01, 0.01, 0.003, 0.006, 0.003)

Initial penalty parameter r_1 = 1.0

Initial objective function OBJ = 1033.33

Upper bound on the standard deviation of kinematic response E = 0.2

Number of iterations required for minimising PF = NITER

K	R_k	NITER	Optimum point $\vec{X_k}$	PF	OBJ
1	1.0×10^{0}	10	(0.103536, 0.106354, 0.0255466, 0.058471, 0.0302326)	174.77	108.38
2	1.0×10^{-1}	10	(0.173676, 0.173575, 0.0283678, 0.0688834,0.0339073)	102.17	90.77
3	1.0×10^{-2}	9	(0.184644, 0.184263, 0.0295779, 0.0730307, 0.0354964)	89.103	86.516
4	1.0×10^{-3}	9	(0.184653, 0.18427, 0.0298759, 0.0730595, 0.0357258)	86.48	85.99

Active constraints are as follows: GG(1) = -0.07673 = Upper bound on Δx

GG(2) = -0.00786 = Upper bound on Δy

GG(3) = -0.00413 = Upper bound on Δr

GG(4) = -0.02587 = Upper bound on Δr_b

GG(5) = -0.027465= Upper bound on Δa

TABLE 3.5 Standard deviation of the kinematic response

Angle ϕ_2	standard deviation corresponding to					Mean kinematic response
	starting point $\vec{X}_o$	$\vec{X}_{opt.}$ for E=0.01	$\vec{X}_{opt.}$ for E=0.06	$\vec{X}_{opt.}$ for E=0.1	$\vec{X}_{opt.}$ for E=0.2	
0	0.4051E-02	0.9426E-02	0.4579E-01	0.6120E-01	0.6721E-01	0.0000E+00
5	0.4057E-02	0.9452E-02	0.4585E-01	0.6125E-01	0.6726E-01	0.9726E-03
10	0.4074E-02	0.9522E-02	0.4605E-01	0.6142E-01	0.6742E-01	0.7730E-02
15	0.4102E-02	0.9638E-02	0.4637E-01	0.6169E-01	0.6764E-01	0.2580E-01
20	0.4138E-02	0.9781E-02	0.4679E-01	0.6204E-01	0.6795E-01	0.6023E-01
25	0.4173E-02	0.9914E-02	0.4718E-01	0.6238E-01	0.6825E-01	0.1153E+00
30	0.4194E-02	0.9988E-02	0.4738E-01	0.6260E-01	0.6844E-01	0.1945E+00
35	0.4198E-02	0.9989E-02	0.4736E-01	0.6267E-01	0.6849E-01	0.3002E+00
40	0.4209E-02	0.9990E-02	0.4745E-01	0.6281E-01	0.6860E-01	0.4335E+00
45	0.4222E-02	0.9990E-02	0.4757E-01	0.6298E-01	0.6875E-01	0.5945E+00
50	0.4232E-02	0.9979E-02	0.4765E-01	0.6313E-01	0.6886E-01	0.7820E+00
55	0.4237E-02	0.9959E-02	0.4766E-01	0.6322E-01	0.6933E-01	0.9930E+00
60	0.4237E-02	0.9935E-02	0.4760E-01	0.6325E-01	0.6952E-01	0.1226E+01
65	0.4234E-02	0.9910E-02	0.4750E-01	0.6325E-01	0.6894E-01	0.1474E+01
70	0.4230E-02	0.9889E-02	0.4738E-01	0.6322E-01	0.6890E-01	0.1734E+01
75	0.4224E-02	0.9873E-02	0.4725E-01	0.6318E-01	0.6885E-01	0.2000E+01
80	0.4219E-02	0.9864E-02	0.4713E-01	0.6313E-01	0.6880E-01	0.2266E+01
85	0.4213E-02	0.9861E-02	0.4702E-01	0.6307E-01	0.6874E-01	0.2526E+01
90	0.4208E-02	0.9863E-02	0.4692E-01	0.6302E-01	0.6869E-01	0.2774E+01
95	0.4204E-02	0.9869E-02	0.4686E-01	0.6296E-01	0.6864E-01	0.3006E+01
100	0.4201E-02	0.9878E-02	0.4682E-01	0.6291E-01	0.6860E-01	0.3218E+01
105	0.4198E-02	0.9889E-02	0.4681E-01	0.6287E-01	0.6857E-01	0.3405E+01
110	0.4197E-02	0.9901E-02	0.4683E-01	0.6284E-01	0.6854E-01	0.3566E+01
115	0.4196E-02	0.9912E-02	0.4687E-01	0.6281E-01	0.6853E-01	0.3700E+01
120	0.4196E-02	0.9922E-02	0.4694E-01	0.6278E-01	0.6852E-01	0.3805E+01
125	0.4194E-02	0.9926E-02	0.4699E-01	0.6274E-01	0.6849E-01	0.3885E+01
130	0.4192E-02	0.9926E-02	0.4704E-01	0.6269E-01	0.6847E-01	0.3940E+01
135	0.4189E-02	0.9925E-02	0.4709E-01	0.6264E-01	0.6843E-01	0.3974E+01
140	0.4187E-02	0.9925E-02	0.4714E-01	0.6259E-01	0.6840E-01	0.3992E+01
145	0.4185E-02	0.9929E-02	0.4720E-01	0.6255E-01	0.6838E-01	0.4000E+01
150	0.4184E-02	0.9938E-02	0.4727E-01	0.6252E-01	0.6837E-01	

TABLE 3.6

Initial feasible point $\vec{X}_1$ = (.003, .006, .003, .00001, 1.0, 20, .0002, .0002, .005, .005)

r_1 = 1.0

PF = 111416.7

OBJ= 111234.4

Upper bound on the standard deviation of the kinematic response E = 0.06

Number of iterations required for minimising PF = NITER

K	R_k	NITER	Optimum point $\vec{X}_k$	PF	OBJ
1	1.0×10^{0}	6	(.0218859, .0110902, .021889, .00001224, 1.049003, 14.8785, .000479471, .000476458, .009330, .003239)	86810.2	86473.4
2	1.0×10^{-1}	6	(.0218859, .0110902, 0.0218895, .00001283, 1.04903, 14.8781, .000479509, .000476497, .0093304, .003239)	82937.1	82681.2
3	1.0×10^{-2}	6	(.0225537, .0137633, .0225861, .00001284, 1.30194, 14.6976, .00049908, .00049965, .00982501, .0032785)	82482.8	82449.4
4	1.0×10^{-3}	6	(.0227330, .0137694, .0225626, .000012894, 1.3025, 14.7173, .000496838, .000499950, .00976892, .0032742)	82162.6	82133.7
5	1.0×10^{-4}	4	(.022733, .0137694, .0225626, .000012895, 1.30252, 14.7192, .000496839, .0004995, .00976892, .00327421)	82135.5	82131.3
6	1.0×10^{-5}				

66(7)=-.00632266 = Upper bound on C_1

66(8)=-.00009953 = Upper bound on C_2

66(51)=-.0010750 (at = 150°)

GG (46)=-.009971 = Upper bound on standard deviation of lift at α = 125°

GG (47)=-.00437 = " " " " α = 130°

GG (48)=-.00132 = " " " " α = 135°

GG (49)=-.0000575= " " " " α = 140°

GG (50)=-.0000983= " " " " α = 145°

TABLE 3.7

Initial feasible point $\vec{X_1}$ = (.003, .006, .003, .00001, 1.0, 20., .0002, .0002, 005, .005)

r_1 = 1.0

PF = 111320.1

OBJ= 111234.4

Upper bound on the standard deviation of the kinematic response E= 0.1

Number of iterations required for minimising PF = NITER

K	R_k	NITER	Optimum point $\vec{X_k}$	PF	OBJ
1	1.0×10^{0}	6	(.0194879, .0282614, .0265828, .00002222, 1.58952, 19.2397, .000388294, .00038829, .0222794, .0195496)	51127.1	50368.3
2	1.0×10^{-1}	6	(0.0194937, .0282642, .0265874, .0000224, 1.61522, 19.2305, .0004979, .0004979, .0222975, .0192338)	49145.9	48879.5
3	1.0×10^{-2}	6	(.0194937, .0282642, .0265874, .00002243, 1.61522, 19.2303, .0004979, .0004979, .0222975, .0192338)	48864.5	48811.2
4	1.0×10^{-3}	6	(.0195433, .0282907, .026619, .000022445, 1.62123, 19.2319, .00049984, .00049813, .0222828, .0192315)	48794.6	48781.6
5	1.0×10^{-4}	5	(.0195502, .028329, .0266615, .000022452, 1.62367, 19.2314, .000499968, .000498138, .0222768, .0192324)	48776.8	48767.7

Active constraints are as follows:

GG (7)=-.00006306916 = Upper bound on C_1
GG (49)=-.0001713112 = " " " " the Std. dev. of lift at α=140°
GG (50)=-0.00008783489= " " " " " " " α=145°
GG (51)=-0.0003837422 = " " " " " " " α=150°

TABLE 3.8

Initial point x_1 = (.003, .006, .003, .00001,1.0,20.0, .0002, .0002,.005, .005)
r_1 = 1.0
PF = 111308.6
OBJ= 111234.4

Upper bound on the standard deviation of the kinematic response E = 0.1

Number of iterations required for minimising PF = NITER

K	R_k	NITER	Optimum point X_k	PF	OBJ
1	1.0×10^{0}	6	(.0237071, .01815, .027039, .0000304662, 1.329922, 18.839, .00047304, .00048630, .0186961, .0186783)	37616.2	37235.6
2	1.0×10^{-1}	4	(.0237079, .0181514, .0270398, .00003056, 1.32946, 18.836, .000493516, .00049928, .0186976, .0186797)	37158.3	36992.8
3	1.0×10^{-2}	6	(.0237079, .0181514, .0270398, .00003058, 1.32940, 18.836, .00049352, .00049929, .0186976, .0186797)	36996.4	36966.1
4	1.0×10^{-3}	6	(.0242586, 0.0190722, .027539, .000030596, 1.44461, 19.034, .00049553, .00049982, .0195693, .019553	36946.4	36935.6
5	1.0×10^{-4}	6	(.0243212, .0191716, .0275968, .000030596, 1.45526, 19.0254, .000495604, .000499954, .019664, .0196481)	36935.8	36933.9
6	1.0×10^{-5}	6	(.0243204, .0191716, .0276026, .0000305999, 1.45634, 18.9994, .000495611, .000499977, .019664, .0196490)	36930.9	36930.0

GG(4) =−0.0000160893 = Upper bound on maximum
GG(7) =−0.00877704 = " " " C_1
GG(8) =−0.0000459254 = " " " C_2

TABLE 3.9 Standard deviation of the dynamic response

Angle ϕ_2	standard deviation corresponding to: starting point $\vec{X}_0$	$\vec{X}_{opt.}$ for E=0.06	$\vec{X}_{opt.}$ for E=0.1	$\vec{X}_{opt.}$ for E=0.2	Mean dynamic response
0	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.0000000
5	0.144233E-03	0.974250E-04	0.555528E-03	0.566798E-03	0.0000908
10	0.580591E-03	0.394498E-03	0.223637E-02	0.228124E-02	0.0009505
15	0.129161E-02	0.885730E-03	0.497293E-02	0.507172E-02	0.0046156
20	0.223815E-02	0.155307E-02	0.859782E-02	0.877025E-02	0.0150091
25	0.336266E-02	0.237637E-02	0.129346E-01	0.131056E-01	0.0380117
30	0.458566E-02	0.334589E-02	0.172403E-01	0.176534E-01	0.0810345
35	0.259469E-02	0.272872E-02	0.677212E-02	0.758918E-02	0.1521145
40	0.404255E-02	0.340938E-02	0.920171E-02	0.108499E-01	0.2588867
45	0.611857E-02	0.674355E-02	0.127619E-01	0.156379E-01	0.4070074
50	0.884688E-02	0.975697E-02	0.174819E-01	0.219992E-01	0.5993085
55	0.122073E-01	0.134313E-01	0.233211E-01	0.298766E-01	0.8348469
60	0.161269E-01	0.176936E-01	0.301615E-01	0.390971E-01	1.1086352
65	0.204804E-01	0.224144E-01	0.377984E-01	0.493679E-01	1.4119171
70	0.251012E-01	0.274171E-01	0.459574E-01	0.603027E-01	1.7329902
75	0.298107E-01	0.325013E-01	0.544167E-01	0.715361E-01	2.0584571
80	0.353196E-01	0.378159E-01	0.698831E-01	0.883189E-01	2.3747120
85	0.388502E-01	0.421349E-01	0.718637E-01	0.938634E-01	2.6694304
90	0.424934E-01	0.461924E-01	0.773209E-01	0.101843E+00	2.9328352
95	0.457368E-01	0.497193E-01	0.829651E-01	0.109450E+00	3.1585496
100	0.484164E-01	0.526235E-01	0.877194E-01	0.115792E+00	3.3439277
105	0.505311E-01	0.549122E-01	0.915002E-01	0.120814E+00	3.4898369
110	0.521304E-01	0.566408E-01	0.943733E-01	0.124620E+00	3.5999736
115	0.532923E-01	0.578942E-01	0.964680E-01	0.127389E+00	3.6798560
120	0.541317E-01	0.588478E-01	0.982111E-01	0.129499E+00	3.7356987
125	0.546732E-01	0.594016E-01	0.991108E-01	0.130760E+00	3.7733798
130	0.550171E-01	0.597372E-01	0.996280E-01	0.131538E+00	3.7976826
135	0.552151E-01	0.599205E-01	0.998884E-01	0.131968E+00	3.8119360
140	0.553000E-01	0.599965E-01	0.999885E-01	0.132146E+00	3.8180922
145	0.552908E-01	0.599941E-01	0.999966E-01	0.132133E+00	3.8171956
150	0.552030E-01	0.599354E-01	0.999667E-01	0.131978E+00	3.8101168

continue on the next page.

TABLE 3.9 Standard deviation of the dynamic response

Angle ϕ_2	standard deviation corresponding to				Mean dynamic response
	starting point $\vec{X}_0$	$\vec{X}_{opt.}$ for $E=0.06$	$\vec{X}_{opt.}$ for $E=0.1$	$\vec{X}_{opt.}$ for $E=0.2$	
155	0.257649E-02	0.263711E-02	0.308831E-02	0.333422E-02	3.8067249
160	0.253958E-02	0.262120E-02	0.307226E-02	0.331475E-02	3.8027814
165	0.249424E-02	0.260503E-02	0.307188E-02	0.329988E-02	3.7934153
170	0.245905E-02	0.259558E-02	0.308924E-02	0.329669E-02	3.7823713
175	0.244754E-02	0.259292E-02	0.309716E-02	0.329675E-02	3.7716857
180	0.246952E-02	0.259735E-02	0.307766E-02	0.329467E-02	3.7633106
185	0.253532E-02	0.261913E-02	0.306852E-02	0.331167E-02	3.7587577
190	0.265144E-02	0.267594E-02	0.315774E-02	0.339108E-02	3.7586255
195	0.280663E-02	0.277370E-02	0.339532E-02	0.355433E-02	3.7634609
200	0.296365E-02	0.288841E-02	0.370879E-02	0.375834E-02	3.7717716
205	0.306910E-02	0.297123E-02	0.394010E-02	0.390833E-02	3.7822063
210	0.307601E-02	0.297569E-02	0.394945E-02	0.391537E-02	3.7728353

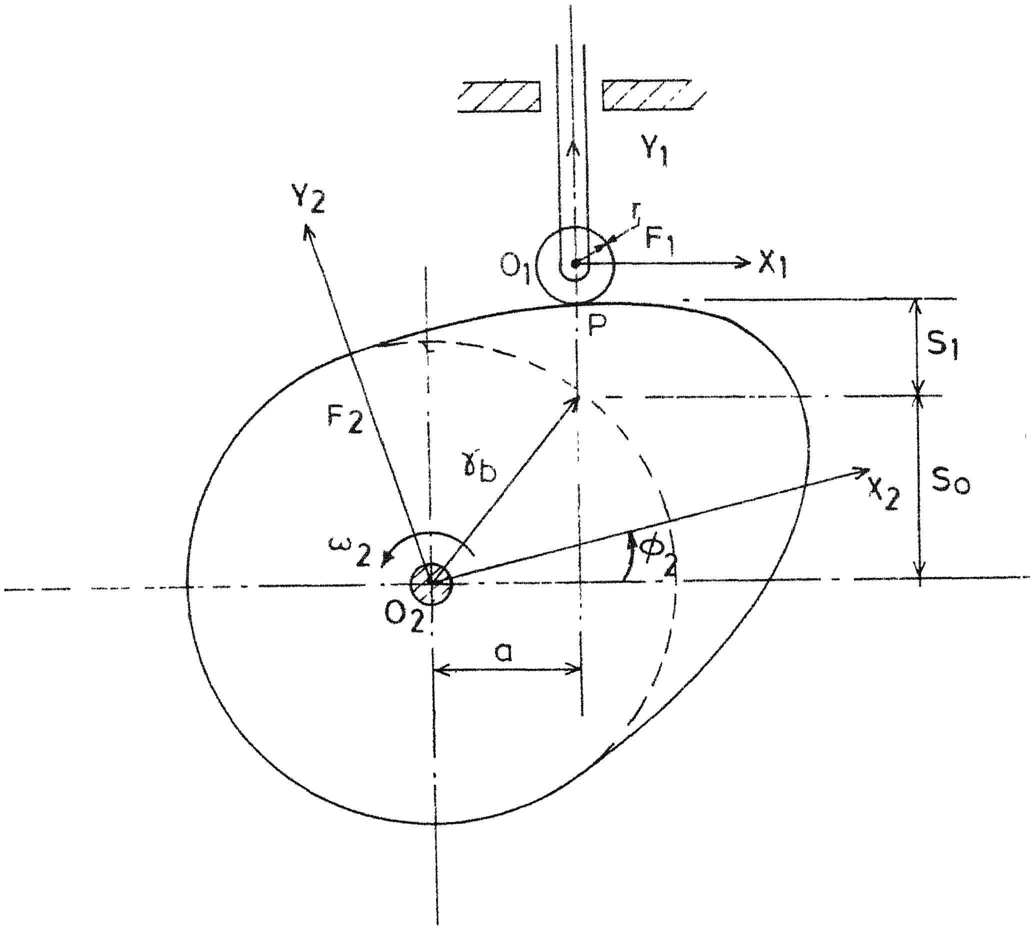


FIG 3.1 DISC CAM WITH TRANSLATING ROLLER FOLLOWER

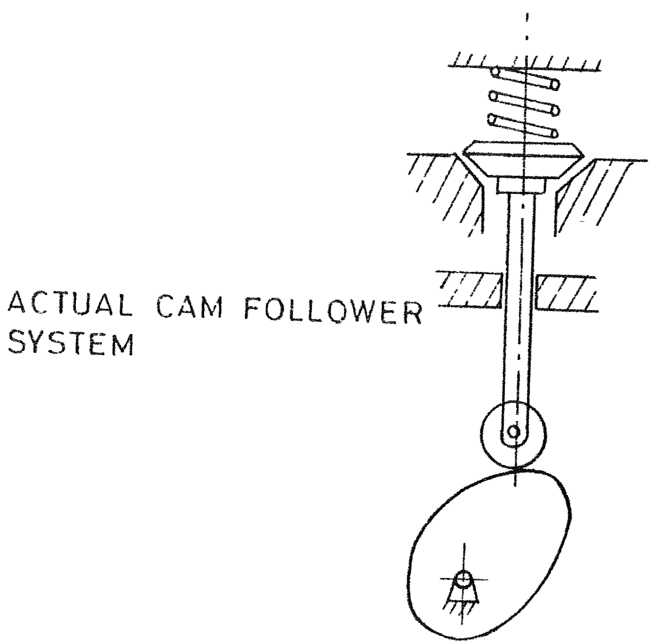


FIG. 3-3.a

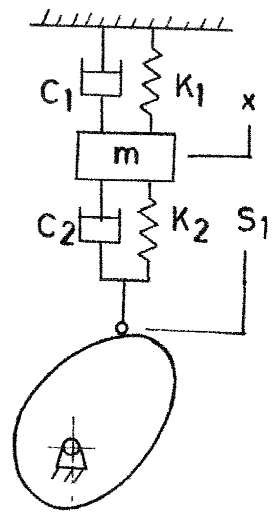


FIG. 3-3.b

FIG.3.3 IDEALISETION OF THE ACTUAL CAM FOLLOWER SYSTEM

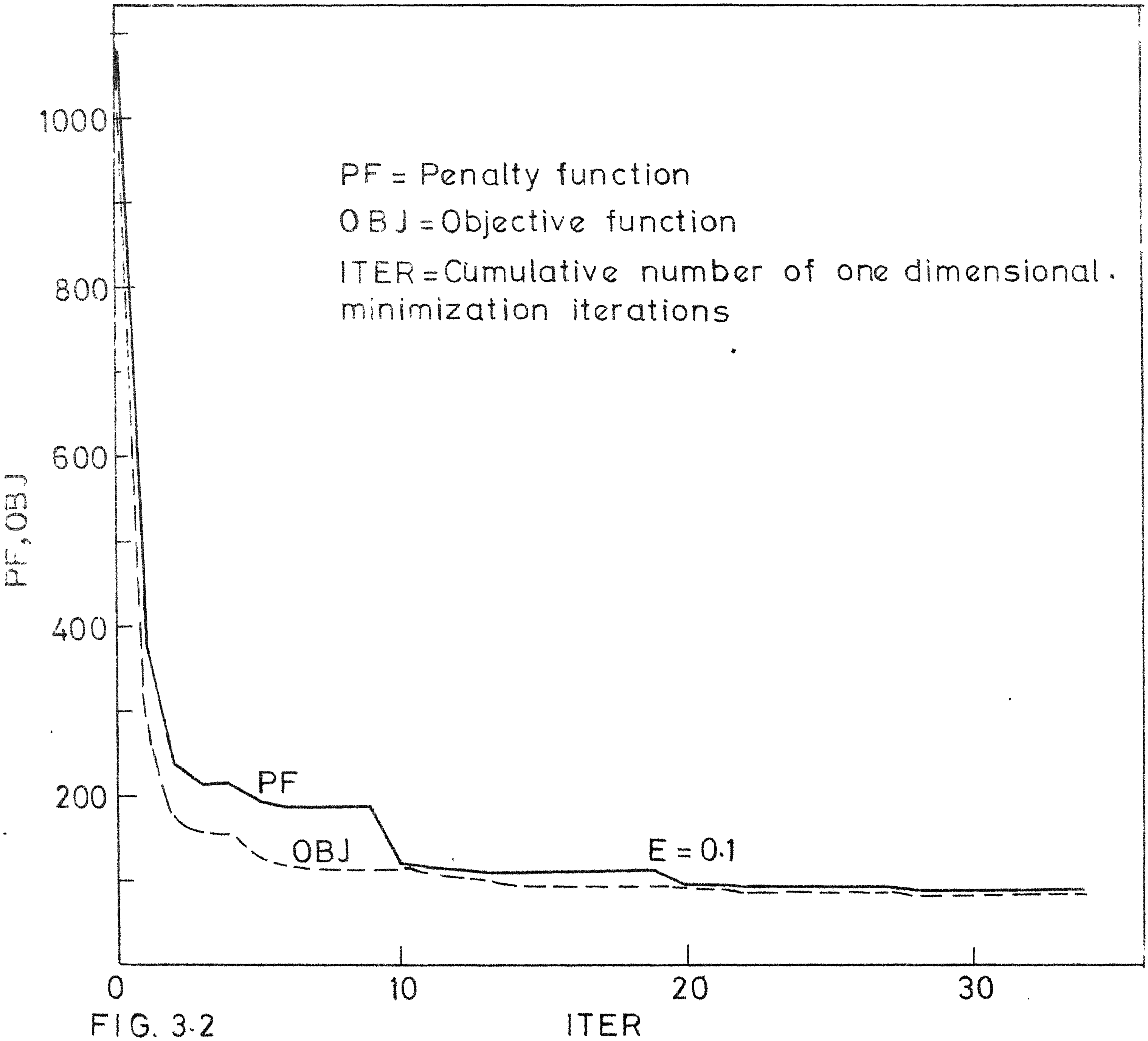


FIG. 3.2

PROGESS OF OPTIMIZATION FOR CAM PROBLEM (Kinematic response)

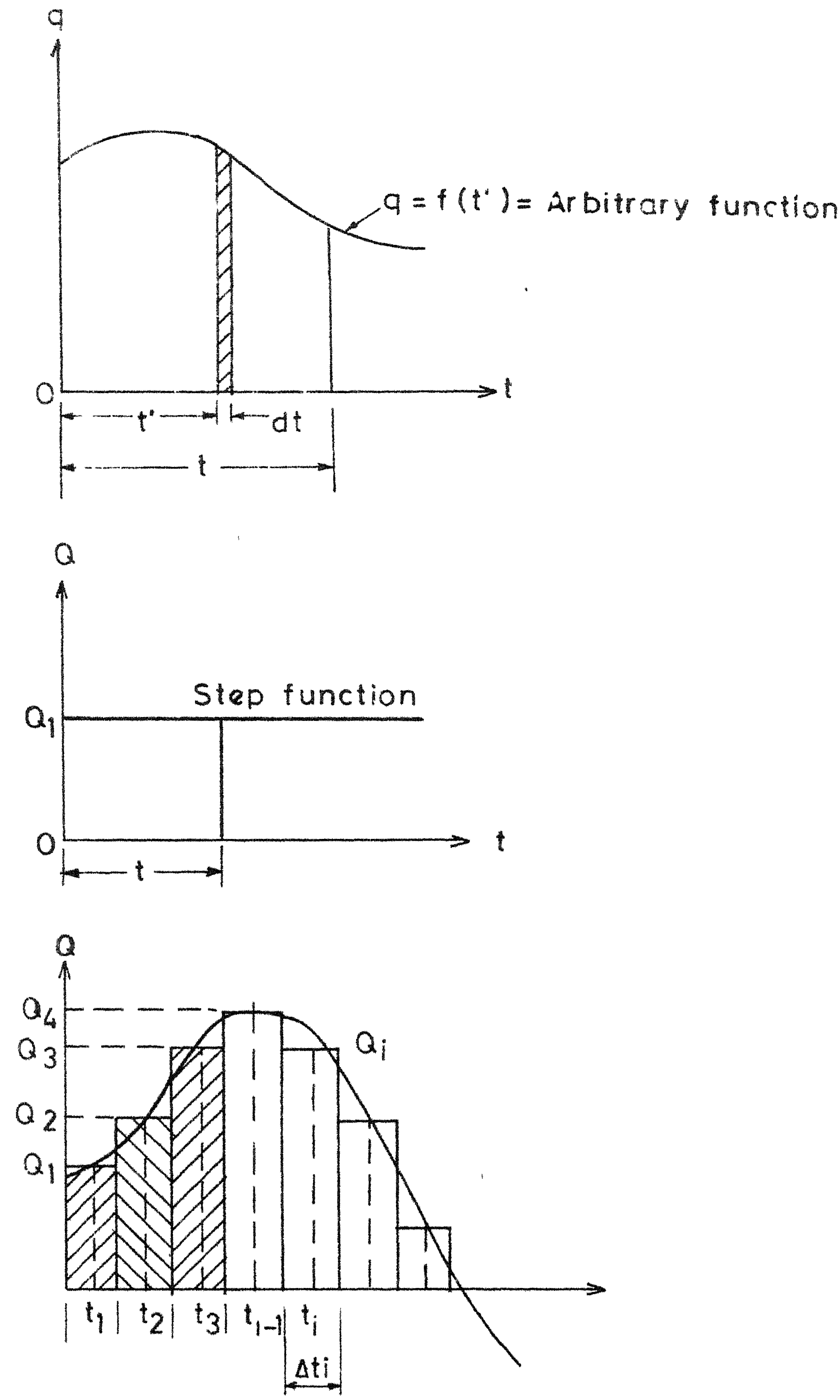
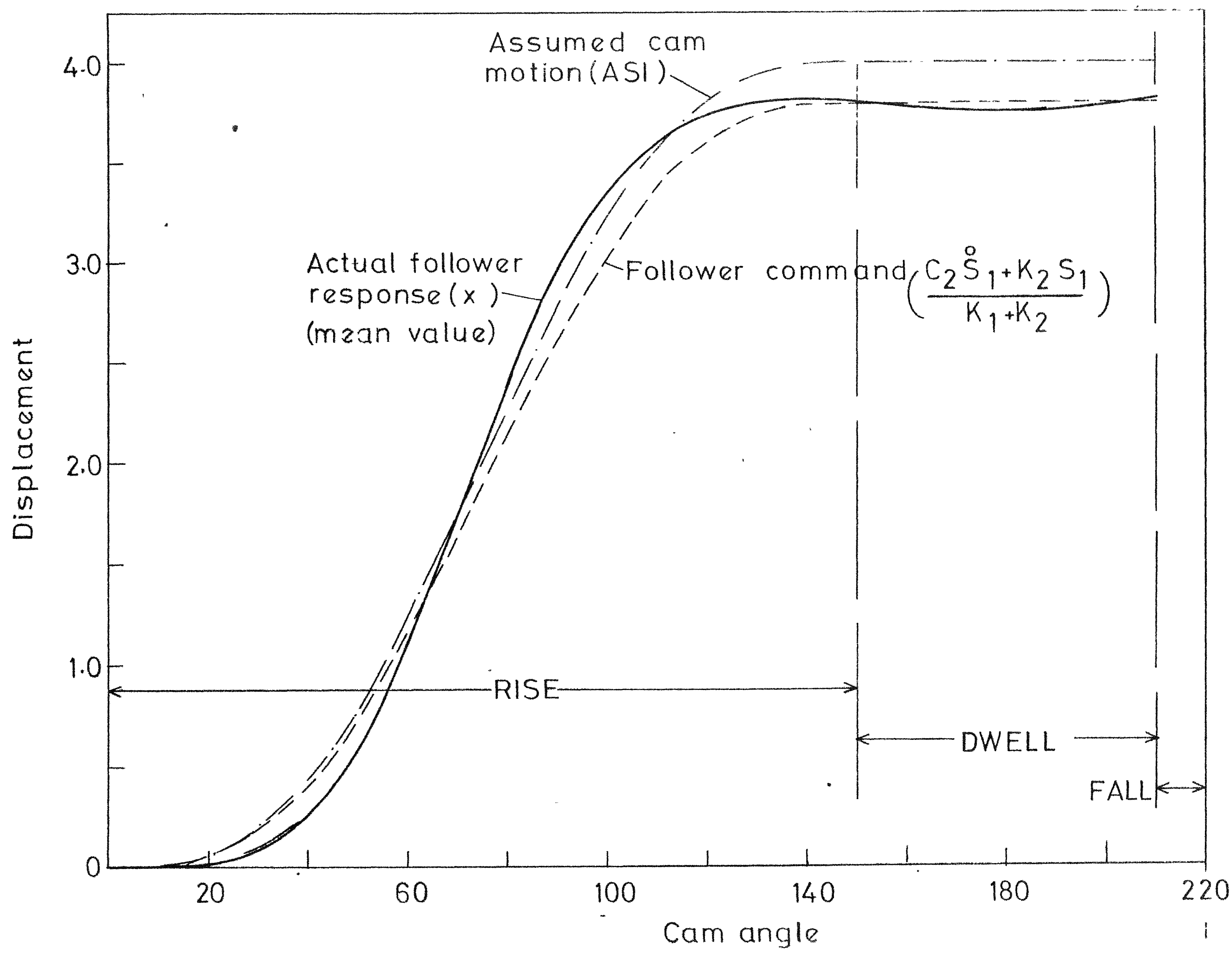


FIG 3.4
APPROXIMATION OF AN ARBITRARY FUNCTION BY A SERIES OF STEP FUNCTIONS


Assumed cam motion (ASI)
Actual follower response (x) (mean value)
Follower command $\left(\frac{C_2 \mathring{S}_1 + K_2 S_1}{K_1 + K_2}\right)$
RISE
DWELL
FALL
Displacement
Cam angle
0
1.0
2.0
3.0
4.0
20
60
100
140
180
220

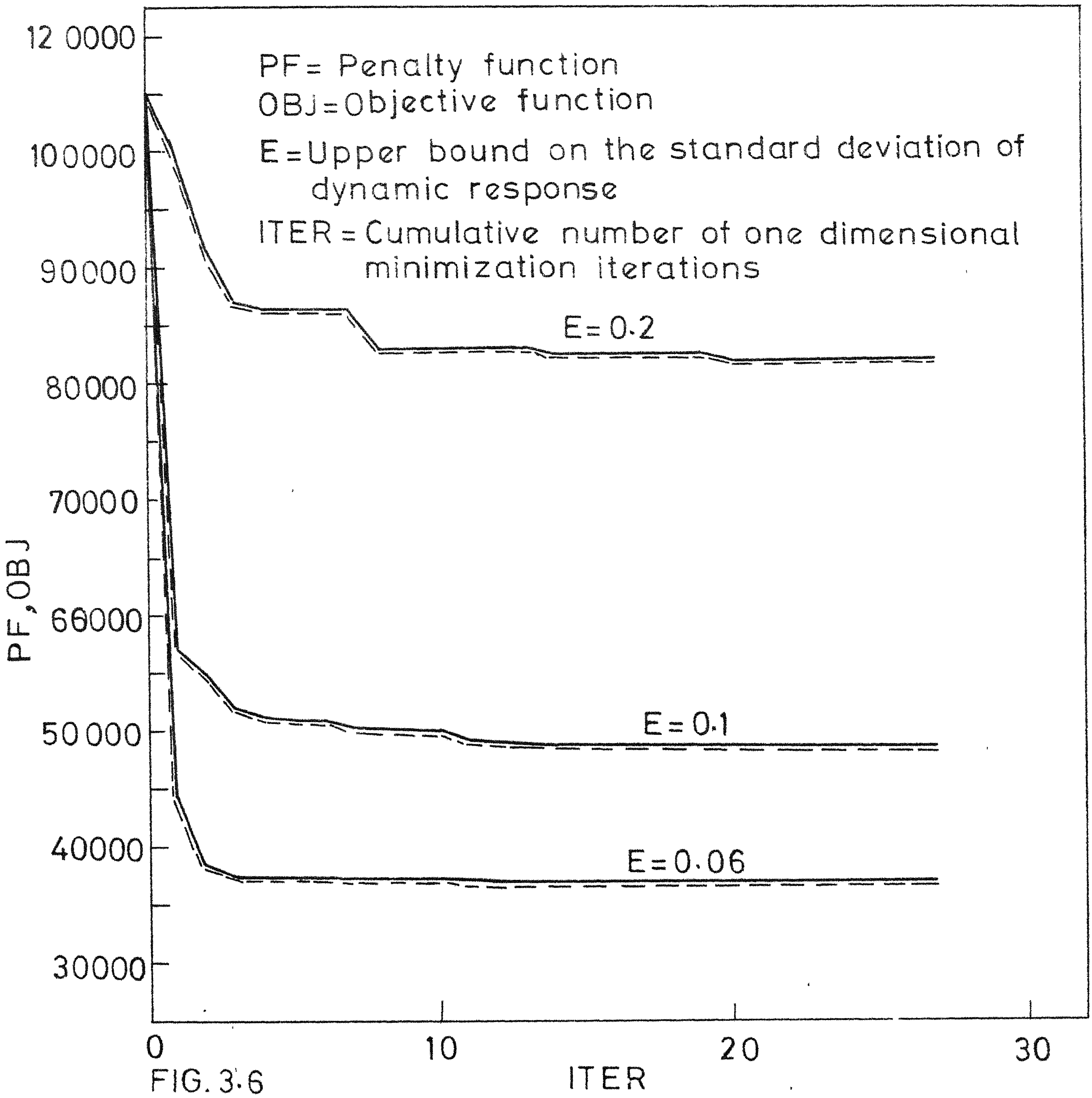


FIG. 3.6

PROGRESS OF OPTIMIZATION FOR CAM PROBLEM

(Dynamic response)

CHAPTER 4

GEAR TRAIN

4.1 PRECISION GEARING:

Gear applications vary so much that the design requirements range from minimum controls and maximum loose tolerances, to complete controls and minimum tolerances of the order of 0.0001 inch. Gears of high quality with tolerances in the order of thousandths and ten-thousandths of an inch, are called precision gears. These occur in all types, pitches, sizes and materials. Precision gearing in its applications is largely limited to instrumentation, automatic controls (including servo-mechanisms), computers, machine tools and special high-speed/or high-power units.

A relatively recent innovation is the specification of stringent performance requirements upon gearing as one component in a complex system. Many precision gears used in the electronics industry and military weapon systems are rigorously assessed for performance and error contribution. The art of gear technology has been transformed to an engineering science encompassing rigorous mathematical analysis and error predicting techniques.

The primary objective of a precision gear is to provide the functions of motion, power transmission and

dynamic response with a high degree of accuracy. In order to achieve the desired level of performance, the various sources of error within the gear train have to be controlled. The two major errors in a gear train are transmission error and backlash. The overall precision of a gear train is often specified by a quantity known as integrated gear train position error, which is the net sum of transmission error and backlash. These errors and their sources are defined in Appendix B. The probabilistic analysis of the various errors was given by Michalec |5| and is summarized in Appendix B. With the availability of nonliner optimization methods, it is possible to optimally allocate the various tolerances (errors) in a gear train while satisfying the specified requirements of precision.

4.2 ALLOCATION OF ERRORS IN A GEAR TRAIN

In general synthesis problem can be stated as an optimization problem as follows:

Find $\vec{X} = (x_1 \quad x_2 \quad x_3 \quad x_4 , \ldots x_n)^T$

which minimizes $OBJ = \sum_{i=1}^{n} \frac{1}{x_i}$

and satisfies the constraints

$$l_i \leq x_i \leq u_i$$

$$\mu_I \leq (\mu_I)_{max.}$$

$$\sigma_I \leq (\sigma_I)_{max.}$$

where x_i, $i = 1, 2, \ldots, x_n$ = errors (tolerances) to be allocated,

l_i, u_i = lower and upper bounds on x_i,

μ_I and σ_I = mean and standard deviations of integrated gear train position error, and

$(\mu_I)_{max.}$ and $(\sigma_I)_{max.}$ = maximum permissible values of μ_I and σ_I.

NUMERICAL EXAMPLE:

The gear train shown in Fig. 4.1 is considered for the optimal allocation of errors. The design vector is taken as

$$X = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{Bmatrix} = \begin{Bmatrix} \text{maximum position error for gears of mesh 1} \\ \text{ball bearing eccentricity for mesh 1} \\ \text{maximum position error for gears of mesh 2} \\ \text{maximum total composite error for gears of mexh 3} \\ \text{centre distance tolerance for mesh 3} \\ \text{gear size tolerance for gears of mesh 3} \\ \text{component shaft runout for mesh 3} \end{Bmatrix}$$

The starting value, the lower and the upper bounds on the design variables are taken as follows (inches):

i	l_i	u_i	(x_i) starting
1	0.00005	0.00100	0.00010
2	0.00005	0.00100	0.00015
3	0.00005	0.00100	0.00015
4	0.00010	0.00100	0.00050
5	0.00010	0.00100	0.00050
6	0.00050	0.00200	0.00100
7	0.00010	0.00100	0.00050

It is assumed that the integrated gear train position error between the encoder shaft S-4 and the output shaft S-1 is to be limited as

$(\mu_I)_{max}$ = 30 seconds of an arc and

$(\sigma_I)_{max}$ = 10 seconds of an arc.

The other pertinent data of the problem is given in Table 4.1.

COMPUTATION OF μ_I and σ_I:

The mean and standard deviations of the integrated gear train position error (μ_I and σ_I) can be computed by using the formulas given in Appendix B as follows:

(a) Transmission error of mesh 1:

Error Sources: 1 Maximum position error

2 Ball bearing eccentricity

The mean and standard deviations of individual gear and pinio. transmission error~ are calculated using Eq. (B.1) and those of mesh 1 using Eq. (B.2). The mesh transmission error is converted into angular measures as+

$$(\mu_T)^{(a)}_{m-1} = \frac{1}{2} (\mu_T)_{m-1} \times \frac{1}{R_1} \left(\frac{180 \times 60 \times 60}{\pi} \right) \text{ secs.}$$

$$(\sigma_T)^{(a)}_{m-1} = \frac{1}{2} (\sigma_T)_{m-1} \times \frac{1}{R_1} \left(\frac{180 \times 60 \times 60}{\pi} \right) \text{ secs.}$$

where R_1 = pitch radius of the gear in mesh 1
= 2.1875 inch.

(b) Backlash of mesh 1:

The backlash error will not be there as spring-loaded antibacklash gears are used in mesh 1. Hence

$$(\mu_B)^{(a)}_{m-1} = (\sigma_B)^{(a)}_{m-1} = 0$$

(c) Transmission error of mesh 2:

Error sources: 1. Maximum pisition error
2. Ball bearing eccentricity
3. Clearance between gear bore and shaft
4. Shaft runout at gear mounting

+ The superscript (a) is used for $(\mu_T)_{m-i}$, $(\sigma_T)_{m-i}$, $(\mu_B)_{m-i}$ and $(\sigma_B)_{m-i}$ to denote angular measures.

Equations (B.1) and (B.2) are used to calculate the mean $(\mu_T)_{m-2}$ and standard deviation $(\sigma_T)_{m-2}$ of mesh 2. These are converted to angular measures as follows:

$$(\mu_T)^{(a)}_{m-2} = \frac{1}{2}(\mu_T)_{m-2} \times \frac{1}{R_2} \times \left(\frac{180 \times 60 \times 60}{\pi}\right) \text{ secs.}$$

$$(\sigma_T)^{(a)}_{m-2} = \frac{1}{2}(\sigma_T)_{m-2} \times \frac{1}{R_2} \times \left(\frac{180 \times 60 \times 60}{\pi}\right) \text{ secs.}$$

where R_2 = pitch radius of the gear = 1.250 inches.

(d) Backlash of mesh 2:

As the gears are spring-loaded,

$$(\mu_B)^{(a)}_{m-2} = (\sigma_B)^{(a)}_{m-2} = 0$$

(e) Transmission error of mesh 3:

Error sources:
1. Total composite error
2. Clearance between gear bore and shaft
3. Shaft runout at gear mounting
4. Ball bearing eccentricity

After determining $(\mu_T)_{m-3}$ and $(\sigma_T)_{m-3}$, the following formulas are used to find their angular measures:

$$(\mu_T)^{(a)}_{m-3} = \frac{1}{2} \times (\mu_T)^{(a)}_{m-3} \times \frac{1}{R_3}\left(\frac{180 \times 60 \times 60}{\pi}\right) \text{ secs.}$$

$$(\sigma_T)^{(a)}_{m-3} = \frac{1}{2} \times (\sigma_T)^{(a)}_{m-3} \times \frac{1}{R_3}\left(\frac{180 \times 60 \times 60}{\pi}\right) \text{ secs.}$$

where the pitch circle radius (R_3) is 1.250 inch.

(f) Backlash of mesh 3:

Error sources: 1. Centre distance tolerance

2. Gear size tolerance

3. Gear size allowance

4. Ball bearing eccentricity

5. Clearance between gear bore and shaft

6. Clearance between bearing outside diameter and housing bore

7. Clearance between shaft and bearing bore

The mean and standard deviations of backlash of mesh 3 are calculated according to Eqs. (B.4) and (B.5). These values are converted to angular measures as

$$(\mu_B)_{m-3}^{(a)} = (\mu_B)_{m-3} \times \frac{1}{R_3} \left(\frac{180 \times 60 \times 60}{\pi}\right) \text{ secs.}$$

$$(\sigma_B)_{m-3}^{(a)} = (\sigma_B)_{m-3} \times \frac{1}{R_3} \left(\frac{180 \times 60 \times 60}{\pi}\right) \text{ secs.}$$

where R_3 = pitch radius of gear = 1.25 inch.

(g) Integrated gear train position error:

The transmission error and backlash between shafts S-1 and S-4 are calculated using Eqs. (B.3) and (B.6) with the velocity ratios to shaft 1 ($V_1 = 1$, $V_2 = 10$, $V_3 = 100$). Finally, the mean and standard deviations of the integrated gear train position error are computed with the help of Eq. (B.7).

NUMERICAL RESULTS

The results of optimization are summarised in Table 4.1. It can be seen that the objective function has been reduced by 32.6%. The upper bounds on the design variables, x_5 (centre distance for mesh 3) and x_7 (component shaft runout for mesh 3) have been found to be active at the optimum point. Among the behaviour constraints, the upper bound on the mean value of the integrated gear train position error has been found to be critical at the final point. The progress of optimization with respect to the number of one dimensional steps is shown in Fig. 4.2.

TABLE 4.1

Design Data for the Gear Train Shown in Fig. 4.1

Item		Mesh Number 1	2	3	4	5
1.	Centre distance tolerance	-	-	$+x_5$ -0	$+0.0005$ -0	$+0.0005$ -0
2.	Gear size tolerance (testing radius)	-	-	$+0$ $-x_6$	$+0$ -0.0005	$+0$ -0.0005
3.	Gear size allowance (testing radius)	-	-	0.0002	0.0002	0.0002
4.	Ball bearing (inner -race) eccentricity	x_2	x_2	x_2	0.0002	0.0002
	Outer race eccentricity	-	-	0.0002	0.0002	0.0002
	Radial play	-	-	0.0003 to 0.0006 ($\mu = 0.00022$, $\sigma = 0.00005$)	0.0003 to 0.0006	0.0003 to 0.0006
5.	Maximum position error tolerance	$\pm x_1$	$\pm x_3$	-	-	-
6.	Maximum total composite error	-	-	x_4	0.0005	0.0005
7.	Shaft run out at gear mounting: gear	-	0.0001	0.0001	See item 11	0.0001
	pinion	-	-	See item 11	0.0001	See item 11

Table contd. on next page

TABLE 4.1 (Continued)

Item		Mesh Number			
	1	2	3	4	5
8. Clearance between gear bore and shaft:		Selected for 0.0001 to 0.0002 Clearance			
bore diameter tolerance	-	μ= 0.00015 σ= 0.00002	+0.0002 -0	0.0003	0.0003
shaft diameter tolerance	-		Gear + 0 -0.0001 Pinion + 0 -0.0002	0.0002	0.0002
allowance	-		0.0001	0.0001	0.0001
9. Clearance between shaft and bearing bore	-	-	0.0001 to 0.0002 (μ=0.000075 σ=0.00001)	0.0001 to 0.0002	0.0001 to 0.0002
10. Clearance between outside diameter and housing bore	-	-	0.0001 to 0.0002 (μ=0.000075 σ=0.00001)	0.0001 to 0.0002	0.0001 to 0.0002
11. Component's tolerance (S-4 and S-6)					
(i) Shaft:					
Shaft run out		x_7 maximum			
Shaft radial play		0.0003 to 0.0006			
Shaft diameter tolerance		0 to - 0.0002			
(ii) Mounting:					
Allowance		0.0001			
12. Pressure angle for all gears ($\emptyset$):		20^{o}			

TABLE 4.2

SUMMARY OF THE RESULTS OF THE OPTIMIZATION PROBLEM OF THE GEAR TRAIN

Starting feasible point $\vec{X_1}$ = (0.0001, 0.00015, 0.00015, 0.0005, 0.0005, 0.001, 0.0005)

Initial OBJ = 30333.33

Initial penalty parameter r_1 = 1.0

Initial PF = 30357.49

Number of iterations required for minimising PF = NITER

K	R_k	NITER	Optimum point $\vec{X_k}$	PF	OBJ
1	$10x10^{\circ}$	10	(0.000162934, 0.000179050, 0.000207804, 0.000832264 0.000925363, 0.000954412, 0.000923470)	21145.07	20947.55
2	$1.0x10^{-1}$	3	(0.000163919, 0.000179754, 0.000208358, 0.000832299, 0.00092539, 0.000957223, 0.000923499)	20924.13	20872.84
3	$1.0x10^{-2}$	10	(0.000164224, 0.000180010, 0.000208709, 0.000832310, 0.000925401, 0.000958092, 0.000923493)	20846.41	20844.49
4	$1.0x10^{-3}$	3	(0.000164285, 0.000180061, 0.000208747, 0.000832311, 0.000925403, 0.000958270, 0.000923495)	20843.99	20839.61
5	$1.0x10^{-4}$	7	(0.000164310, 0.000180097, 0.000208825, 0.000832352, 0.000925348, 0.000958300, 0.000923502)	20837.91	20835.75

Active constraints are as follows

GG(5) = -0.07465207 = Upper bound on x_5

GG(7) = -0.09496763 = Upper bound on x_7

GG(15)=-0.0001391545 =Upper bound on the mean of the total error.

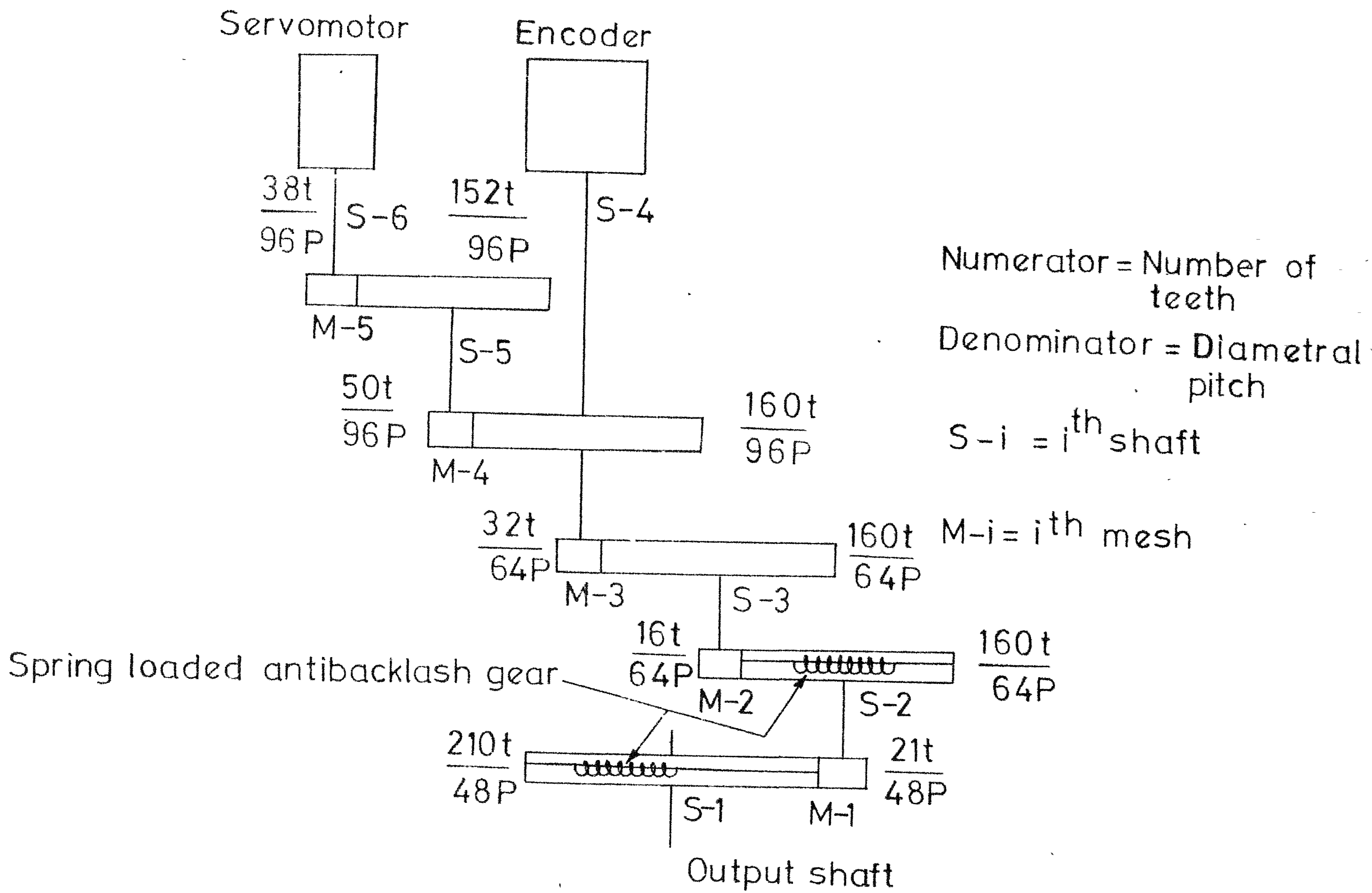


FIG. 4.1 ENCODER GEAR TRAIN

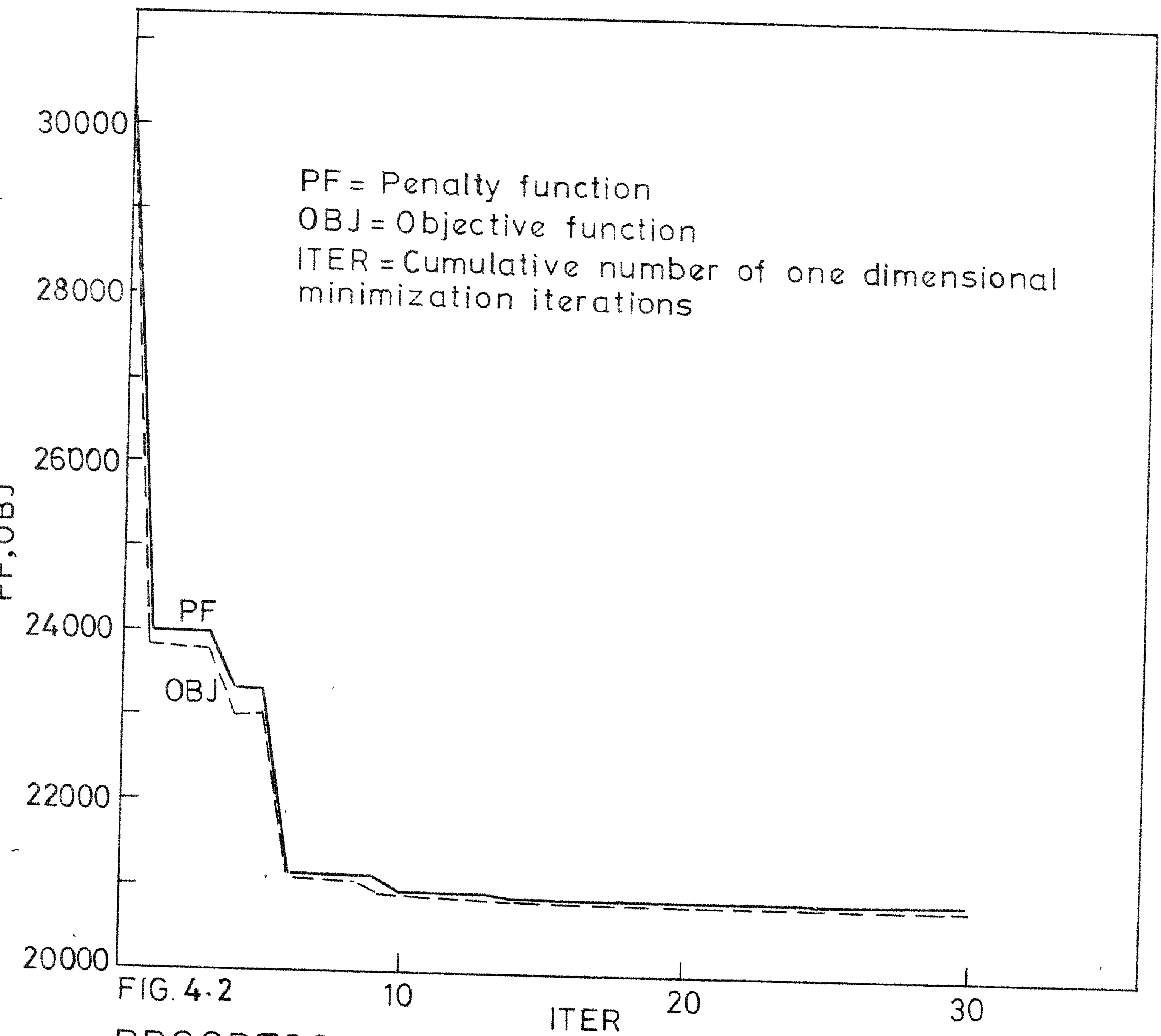


FIG. 4.2

PROGRESS OF OPTIMIZATION FOR GEAR PROBLEM

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

5.1 CONCLUSIONS:

By considering the analysis and synthesis of mechanical error in Geneva mechanisms, cam-follower system and gear train, almost all types of problems in the field of mechanisms have been covered. The probabilistic approach used in this thesis can be considered as a more realistic procedure since, in practice, most of the design parameters are really random in nature. Although normal distribution has been assumed in this work wherever necessary, the basic approach will not change even if the variables follow some other type of distribution.

The tolerances in the centre distance (Δ a) and in the crank length (Δ b) mainly govern the deviation of acceleration and jerk from the mean values in the external Geneva mechanism while the clearance in the slot width (c) controls the change in acceleration and jerk about the mean values in the internal Geneva mechanism. The dimensions (a and b) and the number of slots are same in both the Geneva mechanisms. The initial feasible point is also same. At the optimum point, the objective function gets reduced by 35.2% in the external Geneva mechanism and by 91.3% in the internal Geneva mechanism. So if one of the two

mechanisms is to be used it is cheaper to use the internal Geneva mechanism. In this case and also in other optimization problems solved in this thesis, the major reduction of the objective function occurs in the first two values of the penalty parameter. This shows that if the computer time available is limited, it is advisable to run the optimization programme for only first two values of the penalty parameter and to take the point obtained as an approximation to the true optimum point.

None of the authors in the past combined the kinematic and the dynamic behaviour in the error analysis and synthesis of cam-follower systems. In this work, the response of the follower has been calculated by considering the tolerances in the cam profile, tolerances in the geometrical parameters and errors in other system parameters, like equivalent mass, stiffness and damping during the rise period. It has been revealed that there is no significant improvement in the objective function at the optimum point by relaxing the upper bound on the standard deviation of the kinematic response from 0.06 to 0.2. On the contrary the objective function improves considerably, namely from 26.1% (reduction) to 66.8% (reduction), as the upper bound on the standard deviation of the dynamic response is changed from 0.06 to 0.2. The optimum point, in the case of the kinematic response, is controlled by the upper bounds on the geometrical parameters while in the case of

of dynamic response, the equivalent mass mainly governs the optimum point. An important conclusion drawn is that the kinematic analysis alone is insufficient to define the follower response and that it has to be supplemented by the dynamic response in order to predict the lift of the follower with a greater degree of accuracy.

In the case of the synthesis of gear train, the mean error is not zero as in the cases of Geneva mechanisms and cam follower systems. Here the optimum point is controlled by the upper bound on the mean value of the integrated gear train position error.

5.2 RECOMMENDATIONS:

A more realistic error analysis of the Geneva mechanism problems can be made by considering the frictional force between the slot and the roller.

While calculating the response during the dwell period of the cam follower system, the effect of errors in geometrical parameters was not considered. Hence the present work can be extended by considering the errors in geometrical parameters also, in the dwell period.

It is recommended that the strength based error analysis be coupled with the present error analysis in the synthesis problem of gear-trains.

All the synthesis problems can be solved by posing them in an alternative format as follows:

Find $\vec{X}$ which minimizes

$$OBJ = \sum_{j=1}^{N} (\sigma_{\phi})_j$$

and satisfies the constraints

$$\sum_{i=1}^{n} \frac{1}{x_i} \leq M$$

and $l_i \leq x_i \leq u_i$

where $(\sigma\phi)_j$ is the standard deviation of the output variable ϕ at station j (j = 1, 2, ..., N) and M is an upper bound on the measure of manufacturing cost.

APPENDIX A

PARTIAL DERIVATIVES OF ACCELERATION AND JERK OF GENEVA MECHANISM WITH RESPECT TO THE RANDOM VARIABLES

Let the random variables be V_i ($i = 1$ to 5).

Then $V_1 = X(1) = \Delta a$, $\quad V_2 = X(2) = \Delta b$

$V_3 = X(3) = x_{12}$, $\quad V_4 = X(4) = y_{23}$

$V_5 = X(5) = x_{31}$

The partial derivatives of acceleration and jerk are required to be calculated at the mean design vector. From Eq. (2.33) it can be seen that

$$A = A(a_e, b_e, c_e, \alpha)$$

$$= A(a + \Delta a + x_{12},\ b + \Delta b + x_{31},\ y_{23}, \alpha)$$

$$J = J(a_e, b_e, c_e, \alpha)$$

$$= J(a + \Delta a + x_{12},\ b + \Delta b + x_{31},\ y_{23}, \alpha)$$

The partial derivatives of the equivalent lengths with respect to the various design variables are given below

$$\left.\begin{array}{lll}
\dfrac{\partial a_e}{\partial V_1} = \dfrac{\partial a_e}{\partial V_3} = 1, & \dfrac{\partial a_e}{\partial V_2} = \dfrac{\partial a_e}{\partial V_4} = \dfrac{\partial a_e}{\partial V_5} = 0 \\[2ex]
\dfrac{\partial b_e}{\partial V_2} = \dfrac{\partial v_e}{\partial V_5} = 1, & \dfrac{\partial b_e}{\partial V_1} = \dfrac{\partial b_e}{\partial V_3} = \dfrac{\partial b_e}{\partial V_4} = 0 \\[2ex]
\dfrac{\partial c_e}{\partial V_4} = 1, & \dfrac{\partial c_e}{\partial V_1} = \dfrac{\partial c_e}{\partial V_2} = \dfrac{\partial c_e}{\partial V_5} = 0
\end{array}\right] \quad (A.1)$$

The partial derivative of acceleration with respect to the random variable V_i is given below:

$$\frac{\partial A}{\partial V_i} = \left(\frac{1}{V}\right)\left(\frac{\partial A_1''}{\partial V_i} + \frac{B_1''}{\partial V_i}\right) + \left(\frac{1}{V^3}\right)\left(\frac{\partial A_1}{\partial V_i} + \frac{\partial B_1}{\partial V_i}\right) \cdot \left[(A_1 + B_1)(A_1'' + B_1'') + (A_1' + B_1')^2\right]$$
$$+ \left(\frac{2}{V^3}\right)(A_1 + B_1)(A_1' + B_1')\left(\frac{\partial A_1'}{\partial V_i} + \frac{\partial B_1'}{\partial V_i}\right)$$
$$+ \left(\frac{3}{V^5}\right)(A_1 + B_1)^2 (A_1' + B_1')^2 \left(\frac{\partial A_1}{\partial V_i} + \frac{\partial B_1}{\partial V_i}\right) \quad -(A.2)$$

where $V = \left[1 - (A_1 + B_1)^2\right]^{1/2}$

It can be observed that

$$\left(\frac{\partial A}{\partial V_1}\right)\Big|_{\vec{\underline{X}}} = \left(\frac{\partial A}{\partial V_3}\right)\Big|_{\vec{\underline{X}}}$$

and $\left(\frac{\partial A}{\partial V_2}\right)\Big|_{\vec{\underline{X}}} = \left(\frac{\partial A}{\partial V_5}\right)\Big|_{\vec{\underline{X}}}$

where $\vec{\underline{X}}$ = mean design vector (A.3)

= null vector = (0 , 0, 0, 0, 0)

The partial derivative of jerk with respect to the random variable V_i can be derived as follows:

$$\frac{\partial J}{\partial V_i} = \left(\frac{1}{V}\right)\left(\frac{\partial A_1'''}{\partial V_i} + \frac{\partial B_1'''}{\partial V_i}\right) + \left(\frac{1}{V^3}\right)(A_1 + B_1)(A_1''' + B_1''') \cdot$$
$$\cdot\left(\frac{\partial A_1}{\partial V_i} + \frac{\partial B_1}{\partial V_i}\right) + \left(\frac{9}{V^5}\right)(A_1 + B_1)^2 (A_1' + B_1') \cdot$$
$$\cdot\left(\frac{\partial A_1}{\partial V_i} + \frac{\partial B_1}{\partial V_i}\right) + \left(\frac{3}{V^3}\right)\Big[(A_1 + B_1)(A_1' + B_1').$$

$$\left(\frac{\partial A_1''}{\partial V_i} + \frac{\partial B_1''}{\partial V_i} \right) + (A_1 + B_1) \left(\frac{\partial A_1'}{\partial V_i} + \frac{\partial B_1'}{\partial V_i} \right) (A_1'' + B_1'')$$

$$+ \left(\frac{\partial A_1}{\partial V_i} + \frac{\partial B_1}{\partial V_i} \right)(A_1' + B_1') (A_1'' + B_1'') \Big] + \left(\frac{3}{V^3} \right) (A_1' + B_1')^2$$

$$\left(\frac{\partial A_1'}{\partial V_i} + \frac{\partial B_1'}{\partial V_i} \right) + \left(\frac{3}{V^5} \right)(A_1 + B_1) (A_1' + B_1')^3 \left(\frac{\partial A_1}{\partial V_i} + \frac{\partial B_1}{\partial V_i} \right)$$

$$+ \left(\frac{15}{V^7} \right) (A_1 + B_1)^2 (A_1' + B_1')^3 \left(\frac{\partial A_1}{\partial V_i} + \frac{\partial B_1}{\partial V_i} \right)$$

$$+ \left(\frac{3}{V^5} \right) \Big[2 (A_1 + B_1) (A_1' + B_1')^3 \left(\frac{\partial A_1}{\partial V_i} + \frac{\partial B_1}{\partial V_i} \right)$$

$$+ 3 (A_1' + B_1')^2 (A_1 + B_1)^2 \left(\frac{\partial A_1'}{\partial V_i} + \frac{\partial B_i}{\partial V_i} \right) \Big] \qquad \text{– (A.4)}$$

where $$V = \left[1 - (A_1 + B_1)^2 \right]^{1/2} \qquad \text{(A.5)}$$

As in the case of acceleration, the following relations hold good in the case of jerk also:

$$\left(\frac{\partial J}{\partial V_1} \right) \Big|_{\vec{X}} = \left(\frac{\partial J}{\partial V_3} \right) \Big|_{\vec{X}}$$

$$\text{and} \quad \left(\frac{\partial J}{\partial V_2} \right) \Big|_{\vec{X}} = \left(\frac{\partial J}{\partial V_5} \right) \Big|_{\vec{X}} \qquad \text{(A.6)}$$

A.1 EXTERNAL GENEVA MECHANISM:

From Eq. (2.6), it can be seen that

$$A_1 = \frac{b_e \sin \alpha}{(a_e^2 + b_e^2 - 2\, a_e\, b_e \cos \alpha)^{1/2}}$$

Hence $$(A_1)\Big|_{\vec{X}} = \frac{b \sin \alpha}{(a^2 + b^2 - 2\, ab \cos \alpha)^{1/2}} \qquad \text{(A.7)}$$

Thus, to calculate B_1, B_1', B_1'', B_1''', A_1, A_1', A_1'' and A_1''' at the mean design vector $\overrightarrow{\underline{X}}$, it is necessary to change a_e to a, b_e to b and c_e to zero, in Eqs. (2.6) to Eq. (2.16). The partial derivatives of A_1, A_1', A_1'', A_1''', B_1, B_1', B_1'' and B_1''' with respect to the random variable V_i are given below:

$$\frac{\partial A_1}{\partial V_i} = \left(\frac{1}{Y}\right)\left(\frac{\partial b_e}{\partial V_i} \cdot \text{Sin} \quad\right) - \left(\frac{1}{2\,Y^3}\right) PQ \cdot b_e \,\text{Sin} \tag{A.8}$$

where

$$Y = \left(a_e^2 + b_e^2 - 2\, a_e\, b_e \,\text{Cos}\,\alpha\right)^{1/2} \tag{A.9}$$

$$PQ = \left[2\, a_e \frac{\partial a_e}{\partial V_i} + 2\, b_e \frac{\partial b_e}{\partial V_i} - 2\,\text{Cos}\,\alpha \left(a_e \frac{\partial b_e}{\partial V_i} + b_e \frac{\partial a_e}{\partial V_i}\right)\right] \tag{A.10}$$

$$\frac{\partial B_1}{\partial V_i} = \left(\frac{1}{Y^2}\right)\left[\frac{\partial c_e}{\partial V_i}\left(a_e - b_e \,\text{Cos}\,\alpha\right) + c_e\left(\frac{\partial a_e}{\partial V_i} - \frac{\partial b_e}{\partial V_i}\,\text{Cos}\,\alpha\right)\right] - \left(\frac{1}{Y^4}\right) c_e \left(a_e - b_e\,\text{Cos}\,\alpha\right) \cdot PQ \tag{A.11}$$

$$\frac{\partial A_1'}{\partial V_i} = \left(\frac{1}{Y}\right)\frac{\partial b_e}{\partial V_i}\,\text{Cos}\,\alpha - \left(\frac{1}{2\,Y^3}\right) \cdot b_e\,\text{Cos}\,\alpha \cdot PQ - \left(\frac{1}{Y^3}\right)\text{Sin}^2\alpha\left(\frac{\partial a_e}{\partial V_i}\, b_e^2 + 2\, b_e \frac{\partial b_e}{\partial V_i}\, a_e\right) + \left(\frac{1}{Y^5}\right) \cdot \frac{3}{2}\, a_e\, b_e^2\, \text{Sin}^2\alpha \cdot PQ \tag{A.12}$$

$$\frac{\partial B_1'}{\partial V_i} = \left(\frac{1}{Y^2}\right)\left(\frac{\partial c_e}{\partial V_i} b_e + c_e \frac{\partial b_e}{\partial V_i}\right) \operatorname{Sin} \alpha$$

$$- \left(\frac{1}{Y^4}\right) b_e\, c_e \operatorname{Sin} \alpha \quad PQ - \left(\frac{1}{Y^4}\right) 2 \operatorname{Sin} \alpha \,.$$

$$\Big[(a_e - b_e \operatorname{Cos} \alpha)\left(a_e\, b_e \frac{\partial c_e}{\partial V_i} + a_e\, c_e \frac{\partial b_e}{\partial V_i}\right.$$

$$\left. + b_e\, c_e \frac{\partial a_e}{\partial V_i}\right) + a_e\, b_e\, c_e \left(\frac{\partial a_e}{\partial V_i} - \frac{\partial b_e}{\partial V_i} \cos \quad \right)\Big]$$

$$- \left(\frac{1}{Y^6}\right) . \ 2\, a_e\, b_e\, c_e \operatorname{Sin} \alpha \ (a_e - b_e \operatorname{Cos} \quad)(-2)\ PQ$$

(A.13)

$$\frac{\partial A_1''}{\partial V_i} = - \left(\frac{1}{Y}\right) \frac{\partial b_e}{\partial V_i} \operatorname{Sin} \alpha + \left(\frac{1}{2\, Y^3}\right) . \ b_e \operatorname{Sin} \alpha \ . \ PQ$$

$$- \left(\frac{1}{Y^3}\right) 3 \operatorname{Sin} \alpha \ . \operatorname{Cos} \alpha \left[\frac{\partial a_e}{\partial V_i} b_e^2 + 2\, a_e\, b_e \ \frac{\partial b_e}{\partial V_i}\right]$$

$$- \left(\frac{1}{Y^5}\right) . \ 3\, a_e\, b_e^2 \operatorname{Sin} \alpha \ . \operatorname{Cos} \alpha \ . \left(-\frac{3}{2}\right) . \ PQ$$

$$\cdot \left(\frac{1}{Y^5}\right) 3 \operatorname{Sin}^3 \alpha \left[2\, a_e\, b_e^3 \frac{\partial a_e}{\partial V_i} + 3\, b_e^2\, a_e^2 \frac{\partial b_e}{\partial V_i}\right]$$

$$+ \left(\frac{1}{Y^7}\right) 3\, a_e^2\, b_e^3 \ \operatorname{Sin}^3 \alpha \ \left(-\frac{5}{2}\right) . \ PQ \qquad \text{(A.14)}$$

$$\frac{\partial B_1''}{\partial V_i} = \left(\frac{1}{Y^2}\right)\left(\frac{\partial b_e}{\partial V_i} c_e + \frac{\partial c_e}{\partial V_i} b_e\right) \operatorname{Cos} \alpha$$

$$+ \left(\frac{1}{Y^4}\right) b_e\, c_e \operatorname{Cos} \alpha \ (-1)\ PQ$$

$$- \left(\frac{1}{Y^5}\right) \Big[4\, a_e\, b_e^2\, c_e \operatorname{Sin}^2 \alpha + 2\, a_e\, b_e\, c_e \operatorname{Cos} \alpha \,.$$

$$(a_e - b_e \operatorname{Cos} \alpha)\Big] (-2) . \ PQ$$

$$- \left(\frac{1}{Y^4}\right) 4 \sin^2\alpha \left(\frac{\partial a_e}{\partial V_i} b_e^2 c_e + \frac{\partial c_e}{\partial V_i} a_e b_e^2 \right.$$

$$\left. + 2 a_e b_e c_e \frac{\partial b_e}{\partial V_i} \right) + \left(\frac{2 \cos\alpha}{Y^4} \right) \Big[a_e b_e c_e \cdot$$

$$\left(\frac{\partial a_e}{\partial V_i} - \frac{\partial b_e}{\partial V_i} \cos\alpha \right) + (a_e - b_e \cos\alpha) \cdot$$

$$\left(a_e b_e \frac{\partial c_e}{\partial V_i} + a_e c_e \frac{\partial b_e}{\partial V_i} + b_e c_e \frac{\partial a_1}{\partial V_i} \right) \Big]$$

$$+ \left(\frac{1}{Y^8} \right) . 8 a_e^2 b_e^2 c_e \sin^2\alpha \; (a_e - b_e \cos\alpha)(-3) \; PQ$$

$$+ \left(\frac{1}{Y^6} \right) . 8 \sin^2\alpha \Big[(a_e - b_e \cos\alpha)$$

$$\left(a_e^2 b_e^2 \frac{\partial c_e}{\partial V_i} + 2 a_e b_e^2 c_e \frac{\partial a_1}{\partial V_i} + 2 a_e^2 b_e c_e \frac{\partial b_1}{\partial V_i} \right)$$

$$+ a_e^2 b_e^2 c_e \left(\frac{\partial a_e}{\partial V_i} - \frac{\partial b_e}{\partial V_i} \cos\alpha \right) \Big] \qquad (A.15)$$

$$\frac{\partial A_i'''}{\partial V_i} = - \left(\frac{1}{Y} \right) \frac{\partial b_e}{\partial V_i} \cos\alpha + \left(\frac{1}{Y^3} \right) (-b_e \cos\alpha) \left(- \frac{1}{2} \right) . \; PQ$$

$$+ \left(\frac{1}{Y^5} \right) \left(- \frac{3}{2} \right) . \; PQ \; . \; a_e b_e^2 \left(4 \sin^2\alpha - 3 \cos^2\alpha \right)$$

$$+ \left(\frac{1}{Y^3} \right) . \left(4 \sin^2\alpha - 3 \cos^2\alpha \right) \left(\frac{\partial a_e}{\partial V_i} . \; b_e^2 \right.$$

$$\left. + 2 b_e a_e \frac{\partial b_e}{\partial V_i} \right) + \left(\frac{1}{Y^7} \right) \left(- \frac{5}{2} \right) . \; PQ \; 18 a_e^2 b_e^3 \sin^2\alpha \cos\alpha$$

$$+ \left(\frac{1}{Y^5} \right) . \; 18 \sin^2\alpha \cos\alpha \left(2 a_e \frac{\partial a_e}{\partial V_i} . \; b_e^3 \right.$$

$$\left. + 3 b_e^2 a_e^2 \frac{\partial b_e}{\partial V_i} \right) + \left(\frac{1}{Y^9} \right) \left(\frac{7}{2} \right) PQ \; . \; 15 a_e^3 b_e^4 \sin^4\alpha$$

$$- \left(\frac{1}{Y^7} \right) 15 \sin^4\alpha \left(3 a_e^2 b_e^4 \frac{\partial a_e}{\partial V_i} + 4 b_e^3 a_e^3 \frac{\partial b_e}{\partial V_i} \right) \qquad (A.16)$$

$$
\begin{aligned}
\frac{\partial B_1'''}{\partial V_i} = & - \left(\frac{1}{Y^2} \right) \operatorname{Sin} \alpha \left(\frac{\partial b_e}{\partial V_i} c_e + b_e \frac{\partial c_e}{\partial V_i} \right) \\
& + \left(\frac{1}{Y^4} \right) 2 \operatorname{Sin} \alpha \Big[a_e\, b_e\, c_e \left(\frac{\partial a_e}{\partial V_i} - 7 \frac{\partial b_e}{\partial V_i} \operatorname{Cos} \alpha \right) \\
& + (a_e - 7\, b_e \operatorname{Cos} \alpha) \left(\frac{\partial a_e}{\partial V_i} b_e\, c_e + \frac{\partial b_e}{\partial V_i} a_e\, c_e \right. \\
& \left. + \frac{\partial c_e}{\partial V_i} a_e\, b_e \right) \Big] + \left(\frac{1}{Y^6} \right) . \, 24 \operatorname{Sin} \alpha \operatorname{Cos} \alpha \\
& \Big[a_e^2\, b_e^2\, c_e \left(\frac{\partial a_e}{\partial V_i} - \frac{\partial b_e}{\partial V_i} \operatorname{Cos} \alpha \right) + (a_e - b_e \operatorname{Cos} \alpha) \\
& \left(2\, a_e \frac{\partial a_e}{\partial V_i} b_e^2\, c_e + 2\, b_e \frac{\partial b_e}{\partial V_i} a_e^2\, c_e + a_e^2\, b_e^2 \frac{\partial c_e}{\partial V_i} \right) \Big] \\
& + \left(\frac{1}{Y^6} \right) . \, 24 \operatorname{Sin}^3 \alpha \left(2\, a_e \frac{\partial a_e}{\partial V_i} b_e^3\, c_e + a_e^2 \, . \, 3\, b_e^2 \right. \\
& \left. \frac{\partial b_e}{\partial V_i} c_e + a_e^2\, b_e^3 \frac{\partial c_e}{\partial V_i} \right) - \left(\frac{1}{Y^8} \right) . \, 48 \operatorname{Sin}^3 \alpha \, . \\
& \Big[a_e^3\, b_e^3\, c_e \left(\frac{\partial a_e}{\partial V_i} - \frac{\partial b_e}{\partial V_i} \operatorname{Cos} \alpha \right) \\
& + (a_e - b_e \operatorname{Cos} \alpha) \left(3\, a_e^2 \frac{\partial a_e}{\partial V_i} b_e^3\, c_e \right. \\
& \left. + a_e^3 \, . \, 3\, b_e^2 \frac{\partial b_e}{\partial V_i} c_e + a_e^3\, b_e^3 \frac{\partial c_e}{\partial V_i} \right) \Big] \qquad \text{(A.17)} \\
& + (\text{terms directly multiplied by } c_e)
\end{aligned}
$$

The partial derivative, given by the Eqs. (A.8) to (A.17) are calculated at the mean design vector $\vec{\underline{X}}$ (that is by putting $a_e = a$, $b_e = b$ and $c_e = 0$.)

A.2 INTERNAL GENEVA MECHANISM:

In order to calculate $\left(\frac{\partial A}{\partial V_i} \right)_{\vec{\underline{X}}}$, it is necessary to calculate all the quantities on the right hand side

at the mean design vector $\vec{\underline{X}}$. From Eq. (2.20), it can be seen that

$$A_1 = \frac{b_e \operatorname{Sin} \alpha}{(a_e^2 + b_e^2 + 2\, a_e\, b_e \operatorname{Cos} \alpha\,)^{1/2}} \qquad (A.18)$$

Hence,

$$(A_1)\Big|_{\vec{\underline{X}}} = \frac{b \operatorname{Sin} \alpha}{(a^2 + b^2 + 2\, ab \operatorname{Cos} \alpha\,)} \qquad (A.19)$$

Thus, to evaluate B_1, B_1', B_1'', B_1''', A_1, A_1', A_1'' and A_1''' at the mean design vector $\vec{\underline{X}}$ it is necessary to change a_e to a, b_e to b and c_e to zero. The partial derivatives of B_1, B_1', B_1'', B_1''', A_1, A_1', A_1'' and A_1''' with respect to the random variable V_i are given below.

$$\frac{\partial A_1}{\partial V_i} = (\frac{1}{Y}) \frac{\partial b_e}{\partial V_i} \operatorname{Sin} \alpha - (\frac{1}{2\, Y^3}) \,.\, PQ \,.\, b_e \operatorname{Sin} \alpha \qquad (A.20)$$

where

$$Y = (a_e^2 + b_e^2 + 2\, a_e\, b_e \operatorname{Cos} \alpha\,)^{1/2} \qquad (A.21)$$

$$PQ = 2\, a_e \frac{\partial a_e}{\partial V_i} + 2\, b_e \frac{\partial b_e}{\partial V_i} + 2 \operatorname{Cos} \alpha\, (a_e \frac{\partial b_e}{\partial V_i} + b_e \frac{\partial a_e}{\partial V_i}) \qquad (A.22)$$

$$\frac{\partial B_1}{\partial V_i} = (\frac{1}{Y^2}) \frac{\partial c_e}{\partial V_i} (a_e + b_e \operatorname{Cos} \alpha\,) + c_e\, (\frac{\partial a_e}{\partial V_i} + \frac{\partial b_e}{\partial V_i} \operatorname{Cos} \alpha\,) + (\frac{1}{Y^4})\, c_e\, (a_e + b_e \operatorname{Cos} \alpha\,)\, (-1)\;\; PQ \qquad (A.23)$$

$$\frac{\partial A_1'}{\partial V_i} = \left(\frac{1}{Y}\right)\left(\frac{\partial b_e}{\partial V_i}\right) \mathrm{Cos}\,\alpha + \left(\frac{1}{Y^3}\right)\left(-\frac{1}{2}\right) b_e \,\mathrm{Cos}\,\alpha \; PQ$$

$$+ \left(\frac{1}{Y^3}\right)\left(\frac{\partial a_e}{\partial V_i} b_e^2 + 2\, b_e \frac{\partial b_e}{\partial V_i} a_e\right) \mathrm{Sin}^2\alpha$$

$$+ \left(\frac{1}{Y^5}\right) a_e\, b_e^2\, \mathrm{Sin}^2\alpha \quad \left(-\frac{3}{2}\right) PQ \tag{A.24}$$

$$\frac{\partial B_1'}{\partial V_i} = -\left(\frac{1}{Y^2}\right)\left(\frac{\partial c_e}{\partial V_i} b_e + c_e \frac{\partial b_e}{\partial V_i}\right) \mathrm{Sin}\,\alpha$$

$$+ \left(\frac{1}{Y^4}\right) c_e\, b_e\, \mathrm{Sin}\,\alpha \,.\, PQ$$

$$+ \left(\frac{1}{Y^6}\right) 2\, a_e\, b_e\, c_e\, \mathrm{Sin}\,\alpha \quad (a_e + b_e \,\mathrm{Cos}\,\alpha)(-2)\, PQ$$

$$+ \left(\frac{1}{Y^4}\right) 2\, \mathrm{Sin}\,\alpha \Big[(a_e + b_e \,\mathrm{Cos}\,\alpha)\left(\frac{\partial a_e}{\partial V_i} b_e\, c_e\right.$$

$$+ \frac{\partial b_e}{\partial V_i} a_e\, c_e + \frac{\partial c_e}{\partial V_i} b_e\, a_e + a_e\, b_e\, c_e \left(\frac{\partial a_e}{\partial V_i}\right.$$

$$\left. + \frac{\partial b_e}{\partial V_i} \mathrm{Cos}\,\alpha \right) \Big] \tag{A.25}$$

$$\frac{\partial A_1''}{\partial V_i} = -\left(\frac{1}{Y}\right) \frac{\partial b_e}{\partial V_i} \mathrm{Sin}\,\alpha + \left(\frac{1}{Y^3}\right) b_e\, \mathrm{Sin}\,\alpha \left(\frac{1}{2}\right) PQ$$

$$+ \left(\frac{1}{Y^3}\right) 3\, \mathrm{Sin}\,\alpha\, \mathrm{Cos}\,\alpha \left(\frac{\partial a_e}{\partial V_i} b_e^2 + a_e\, 2\, b_e \frac{\partial b_e}{\partial V_i}\right)$$

$$+ \left(\frac{1}{Y^5}\right) a_e\, b_e^2\, \mathrm{Sin}^2\alpha \quad \left(-\frac{3}{2}\right) PQ$$

$$+ \left(\frac{1}{Y^7}\right) 3\, a_e^2\, b_e^3\, \mathrm{Sin}^3\alpha \left(-\frac{5}{2}\right) PQ$$

$$+ \left(\frac{1}{Y^5}\right) 3\, \mathrm{Sin}^3\alpha \quad \left(2\, a_e \frac{\partial a_e}{\partial V_i} b_e^3 + a_e^2\, 3\, b_e^2 \frac{\partial b_e}{\partial V_i}\right)$$

$$\tag{A.26}$$

$$\frac{\partial B_1''}{\partial V_i} = -\left(\frac{1}{Y^2}\right) \text{Cos}\,\alpha \left(\frac{\partial b_e}{\partial V_i} c_e + \frac{\partial c_e}{\partial V_i} b_e\right)$$

$$+ \left(\frac{1}{Y^4}\right) b_e\, c_e\, \text{Cos}\,\alpha\ PQ \ + \left(\frac{1}{Y^6}\right)\Big[-4\, a_e\, b_e^2\, c_e\, \text{Sin}^2\alpha$$

$$+ 2\, a_e\, b_e\, c_e\, \text{Cos}\,\alpha\ (a_e + b_e \text{Cos}\,\alpha\ )\Big]\ (-2)\ PQ$$

$$- \left(\frac{4}{Y^4}\right) 4\ \text{Sin}^2\alpha\ \left(\frac{\partial a_e}{\partial V_i} b_e^2\, c_e + 2\, a_e\, b_e\, c_e \frac{\partial b_e}{\partial V_i}\right.$$

$$\left. +\ a_e\, b_e^2 \frac{\partial c_e}{\partial V_i}\right) + \left(\frac{2}{Y^4}\right) \text{Cos}\,\alpha \Big[(a_e + b_e\, \text{Cos}\alpha\ ).$$

$$\left(\frac{\partial a_e}{\partial V_i} b_e\, c_e + \frac{\partial b_e}{\partial V_i} c_e\, a_e + \frac{\partial c_e}{\partial V_i} a_e\, b_e\right)$$

$$+ a_e\, b_e\, c_e \left(\frac{\partial a_e}{\partial V_i} + \frac{\partial b_e}{\partial V_i}\ \text{Cos}\alpha\ \right)\Big]$$

$$+ \left(\frac{1}{Y^8}\right) 8\, a_e^2\, b_e^2\, c_e\ \text{Sin}^2\alpha\ \ (a_e + b_e\, \text{Cos}\alpha\ )(-3)\ PQ$$

$$+ \left(\frac{1}{Y^6}\right) 8\ \text{Sin}^2\alpha\ \Big[(a_e + b_e\ \text{Cos}\,\alpha\ ).$$

$$\left(2\, a_e \frac{\partial a_e}{\partial V_i} b_e^2\, c_e + a_e^2\, 2\, b_e \frac{\partial b_e}{\partial V_i} c_e + a_e^2\, b_e^2 \frac{\partial c_e}{\partial V_i}\right)$$

$$+\ a_e^2\, b_e^2\, c_e \left(\frac{\partial a_e}{\partial V_i} + \frac{\partial b_e}{\partial V_i} \text{Cos}\,\alpha\ \right)\Big] \qquad (A.27)$$

$$\frac{\partial A_1'''}{\partial V_i} = -\left(\frac{1}{Y}\right) \frac{\partial b_e}{\partial V_i} \text{Cos}\alpha\ + \left(\frac{1}{Y^3}\right)\left(\frac{1}{2}\right) PQ\ b_e\ \text{Cos}\,\alpha$$

$$+ \left(\frac{1}{Y^5}\right) (-4\, a_e\, b_e^2\ \text{Sin}^2\alpha\ + 3\, a_e\, b_e^2\ \text{Cos}^2\alpha\ )\left(-\frac{3}{2}\right) PQ$$

$$+ \left(\frac{1}{Y^3}\right) (-4\ \text{Sin}^2\alpha\ + 3\ \text{Cos}^2\alpha\ )\left(\frac{\partial a_e}{\partial V_i} b_e^2\right.$$

$$\left. + 2\, a_e\, b_e \frac{\partial b_e}{\partial V_i}\right) + \left(\frac{1}{Y^7}\right) 18\, a_e^2\, b_e^3\ \text{Sin}^2\alpha\ \ \text{Cos}\,\alpha \left(-\frac{5}{2}\right) PQ$$

$$+ \left(\frac{1}{Y^5}\right) 18 \sin^2\alpha \cos\alpha \left(2 a_e \frac{\partial a_e}{\partial V_i} b_e^3\right.$$

$$\left. + a_e^2\, 3\, b_e^2 \frac{\partial b_e}{\partial V_i}\right) + \left(\frac{1}{Y^9}\right) 15 \sin^4\alpha\ a_e^3\, b_e^4 \left(-\frac{7}{2}\right) PQ$$

$$+ \left(\frac{1}{Y^7}\right) 15 \sin^4\alpha \left(3 a_e^2 \frac{\partial a_e}{\partial V_i} b_e^4 + a_e^3\, 4\, b_e^3 \frac{\partial b_e}{\partial V_i}\right)$$

(A.28)

$$\frac{\partial B_1'''}{\partial V_i} = \left(\frac{1}{Y^2}\right) \sin\alpha \left(\frac{\partial b_e}{\partial V_i} c_e + b_e \frac{\partial c_e}{\partial V_i}\right)$$

$$- \left(\frac{1}{Y^4}\right) 2 \sin\alpha \left(2 a_e \frac{\partial a_e}{\partial V_i} b_e\, c_e + a_e^2\, b_e \frac{\partial c_e}{\partial V_i}\right.$$

$$\left. + a_e^2\, c_e \frac{\partial b_e}{\partial V_i}\right) + \left(\frac{1}{Y^5}\right) 24 \sin\alpha \cos\alpha \Big[a_e^2\, b_e^2\, c_e$$

$$\left(\frac{\partial a_e}{\partial V_i} + \frac{\partial b_e}{\partial V_i} \cos\alpha\right) + (a_e + b_e \cos\alpha)$$

$$\left(2 a_e \frac{\partial a_e}{\partial V_i} b_e^2\, c_e + 2 a_e^2\, b_e \frac{\partial b_e}{\partial V_i} c_e + a_e^2\, b_e^2 \frac{\partial c_e}{\partial V_i}\right)\Big]$$

$$- \left(\frac{1}{Y^6}\right) 24 \sin^3\alpha \left(2 a_e \frac{\partial a_e}{\partial V_i} b_e^3\, c_e + 3 a_e^2\, b_e^2\, c_e \frac{\partial b_e}{\partial V_i}\right.$$

$$\left. + a_e^2\, b_e^3 \frac{\partial c_e}{\partial V_i}\right) + \left(\frac{1}{Y^8}\right) 48 \Big[a_e^3\, b_e^3\, c_e \left(\frac{\partial a_e}{\partial V_i} + \frac{\partial b_e}{\partial V_i} \cos\alpha\right)$$

$$+ (a_e + b_e \cos\alpha)\left(3 a_e^2\, b_e^3\, c_e \frac{\partial a_e}{\partial V_i}\right.$$

$$\left. + a_e^3\, 3\, b_e^2 \frac{\partial b_e}{\partial V_i} c_e + a_e^3\, b_e^3 \frac{\partial c_e}{\partial V_i}\right)\Big] \sin^3\alpha$$

$$- \left(\frac{1}{Y^4}\right) 14 \sin\alpha \cos\alpha \left(\frac{\partial a_e}{\partial V_i} b_e^2\, c_e\right.$$

$$\left. + 2 a_e\, b_e \frac{\partial b_e}{\partial V_i} c_e + a_c\, b_e^2 \frac{\partial c_e}{\partial V_i}\right)$$

+ (terms directly multiplied by c_e) (A.29)

The partial derivatives, given by Eqs. (A.20) to (A.29) are calculated at the mean design vector $\bar{\underline{X}}$ (that is by putting $a_e = a$, $b_e = b$ and $c_e = 0$).

APPENDIX B

PROBABILISTIC ANALYSIS OF GEAR TRAIN ERRORS

B.1 INTRODUCTION:

The primary objective in a precision gear train is to provide the functions of motion, power transmission and dynamic response with a high degree of accuracy and capability. The analysis of gear train performance requires identification of error types, sources and their interrelation to produce a total error. In precision gear trains, the errors have to be controlled in an optimal manner. Two performance criteria or errors completely define the precision of performance of a gear train. These are

(i) transmission error

and (ii) backlash.

(i) Transmission error:

It is the variation in the transmission ratio of a gear pair or train from the ideal nominal value. Although in a gear pair each member has a whole number of teeth, which fixes the transmission ratio in terms of integral numbers of turns, the instantaneous ratio during one revolution is likely to fluctuate slightly. Transmission error is the measure of instantaneous variation from ideal nominal value. This is caused by the net sum of

individual gear position errors and installation runout errors.

(ii) Backlash:

It is the total lost motion for a gear pair or train. It can also be defined as the amount of motion a meshed gear has when its mate is held fixed. It is caused by thinned teeth, enlarged center distance etc.

Sometimes the net combined effect of transmission error and backlash, known as integrated gear train position error, is used as a characteristic of performance of the gear train. It is the net sum of transmission error and backlash and is a measure of total deviation from ideal position of a gear pair or train. The various sources that cause transmission error and backlash are given below:

Transmission error sources	Backlash sources
1. Position error in the individual gears	Gear size tolerance and and allowance
(a) Total composite error (TCE)	Centre distance tolerance and allowance
(b) Supplementary position error (SPE)	Eccentricity of fixed bearing
2. Installation errors	Ball bearing radial play due to clearances
(a) Clearance between gear bore and shaft	Shaft runout at point of gear mounting
(b) Shaft runout at point of gear mounting	
(c) Ball bearing radial play due to clearances etc.	Thermal dimensional changes etc.

B.2 TOLERANCES FOR GEAR TRAIN

Some of the errors that are permitted and are given as tolerances in the design of agear pair or train are defined below:

(i) Total composite error (TCE) is defined as the maximum variation in centre distance as the gear is rolled, intimately meshed with a high quality master gear, on a variable centre distance fixture. This is a measure of the cumulative effects of eccentricity, tooth to tooth variation and profile deviation. Thus TCE can be considered as an overall tolerance and its maximum value is specified as x. The permissible values of TCE according to AGMA standards are given in 10 .

The TCE specification can control center distance variation, but it is incapable of detecting all angular errors. It can reveal tooth-to-tooth profile position errors but not accumulated errors. Total angular transmission error can only be revealed by checking the tooth profile for position error.

(ii) Gear tooth profile position error or simply the position error is the deviation of any position of a gear from the true position it should have. This is illustrated in Fig. B.1. This error does not become apparent until it is mated in an application and until a functional output is observed. However,

an individual gear can be evaluated for position error by determining the degree of departure of its tooth profiles from ideal form and space positions. The measurement of the position error can be either linear or angular. As a linear measure it is the displacement from the ideal position measured along the nominal pitch circle. As an angular measure it is the subtended angle relative to the center of the gear. If the maximum allowable position error is specified as $\pm x$, then any profile point should not deviate from its ideal position in excess of the specified amount, x.

(iii) Centre distance tolerance:

In practice, the gear center distance will vary from the ideal value regardless of the manufacturing method used for the gears. The amount of variation on tolerance is a function of the fabrication method used to establish the gear centers. Tolerance on gear center distance is often specified as plus some value minus zero, applied to the ideal nominal center distance.

(iv) Gear size tolerance:

The tooth thickness must be toleranced to permit realistic fabrication. This tolerance establishes an equivalent possible backlash range. The size

tolerances are specified as $\begin{smallmatrix}+0\\-x\end{smallmatrix}$ and the values of x for various AGMA quality classes are given in 10 .

(v) Fixed bearing eccentricity:

The absolute position of gear centres also depends on the tolerances and allowances given for the installation of bearings. Both ball and sleeve bearings that are fixed in a housing by press fit or clamping contribute to backlash due to the eccentricity that alters the effective center distance. The bearing eccentricity is generally specified as $\begin{smallmatrix}+x\\-0\end{smallmatrix}$.

(vi) Component Shaft runout:

Shafts, especially if long and slender will have a measurable amount of runout due to boaring (bending). (Usually the maximum permissible value (x) of the shaft runout is specified). This influences the actual centre distance of the gears as shown in Fig. B.2.

B.3 COMPUTATION OF MEAN AND STANDARD DEVIATIONS OF INTEGRATED GEAR TRAIN POSITION ERROR:

Once the tolerances specified for the gear train are known, the mean and standard deviations of the toleranced quantities can be established (say, by assuming three sigma values) and the mean and standard deviations of transmission error and backlash can be found as indicated below.

Statistical parameters of transmission error:

(i) Establish mean and standard deviation values for each transmission error source (e_i) as μ_i and σ_i, i = 1, 2, ...

(ii) Find mean and standard deviation of total gear transmission error as

$$\mu_{T_g} = \frac{1}{2}(\pi)^{1/2}\left[\sum(\mu_i^2) + \sum(\sigma_i^2)\right]^{1/2}$$

$$\sigma_{T_g} = \left[1 - \left(\frac{\pi}{4}\right)^2\right]^{1/2}\left[\sum(\mu_i)^2 + \quad (\sigma_i^2)\right]^{1/2} \qquad \text{(B.1)}$$

Compute μ_{T_p} and σ_{T_p} using similar formulas for the total pinion transmission error.

(iii) Calculate mean and standard deviation of total mesh transmission error using the relations

$$(\mu_T)_m = \frac{1}{2}(\pi)^{1/2}\left[\mu_{T_g}^2 + \mu_{T_p}^2 + \sigma_{T_g}^2 + \sigma_{T_p}^2\right]^{1/2}$$

for mesh ratio < 2

$$= \mu_{T_g} + \mu_{T_p} \quad \text{for mesh ratio} \geq 2$$

$$(\sigma_T)_m = \left[1 - \left(\frac{\pi}{4}\right)\right]^{1/2}\left(\mu_{T_g}^2 + \mu_{T_p}^2 + \sigma_{T_g}^2 + \sigma_{T_p}^2\right)^{1/2}$$

for mesh ratio < 2

$$\doteq \left(\sigma_{T_g}^2 + \sigma_{T_p}^2\right)^{1/2} \quad \text{for mesh ratio} \geq 2$$

(B.2)

(iv) Determine the mean and standard deviation of entire gear train transmission error as

$$(\mu_T)_t = \frac{1}{2}(\pi)^{1/2}\left[\sum\left(\frac{(\mu_T)_{m-i}}{V_i}\right)^2 + \sum\left(\frac{(\sigma_T)_{m-i}}{V_i}\right)^2\right]^{1/2} \quad \text{for stage ratio} < 2$$

$$= \sum\left(\frac{(\mu_T)_{m-i}}{V_i}\right) \quad \text{for stage ratio} \geq 2$$

$$(\sigma_T)_t = \left[1 - \left(\frac{\pi}{4}\right)\right]^{1/2}\left[\sum\left(\frac{(\mu_T)_{m-i}}{V_i}\right)^2 + \sum\left(\frac{(\sigma_T)_{m-i}}{V_i}\right)^2\right]^{1/2} \quad \text{for stage ratio} < 2$$

$$= \left[\sum\left(\frac{(\sigma_T)_{m-i}}{V_i}\right)^2\right]^{1/2} \quad \text{for stage ratio} \geq 2 \tag{B.3}$$

where the subscript (m - i) indicates the ith mesh and V_i denotes the stage velocity ratio of ith mesh.

Statistical parameters of backlash:

(i) Establish mean and standard deviation values for each backlash error source (e_i) as μ_i and σ_i, i = 1, 2, ...

(ii) Find the mean and standard deviation of backlash of each gear (μ_{B_g}; σ_{B_g}) and of each pinion (μ_{B_p}; σ_{B_p}) as

$$\mu_{B_g} = \sum \mu_i$$
$$\sigma_{B_g} = \left[\sum \sigma_i^2\right]^{1/2} \tag{B.4}$$

with similar formulas for μ_{B_p} and σ_{B_p}

(iii) Compute the mean and standard deviation of mesh backlash as

$$(\mu_B)_m = \mu_{B_g} + \mu_{B_p}$$
$$(\sigma_B)_m = (\sigma_{B_g}^2 + \sigma_{B_p}^2)^{1/2} \qquad (B.5)$$

(iv) Determine the mean and standard deviation of backlash of entire gear train as

$$(\mu_B)_t = \sum \left(\frac{(\mu_B)_{m-i}}{V_i} \right)$$
$$(\sigma_B)_t = \left[\sum \left(\frac{(\sigma_B)_{m-i}}{V_i} \right)^2 \right]^{1/2} \qquad (B.6)$$

After computing the statistical parameters of transmission error and backlash for the entire gear train, the parameters of the integrated gear train position error (μ_I and σ_I) can be obtained as follows:

$$\mu_I = (\mu_B)_t / 2 + (\mu_T)_t / \text{Cos } \phi$$
$$\sigma_I = \left[(\sigma_B)_t^2 / 4 + (\sigma_T)_t^2 / \text{Cos}^2 \phi \right]^{1/2} \qquad (B.7)$$

where ϕ is the pressure angle.

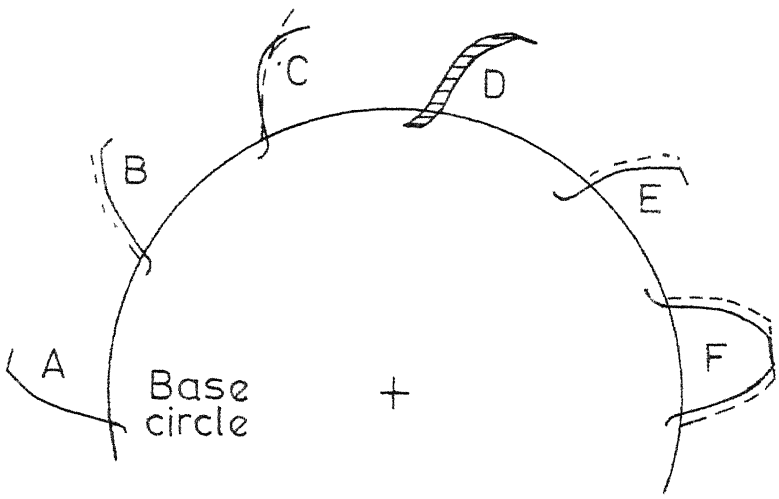


FIG. B.1 EXAMPLES OF TOOTH PROFILE POSITION ERROR. BROKEN PROFILES ARE IDEAL PERFECT-PROFILE POSITIONS, SHOWN FOR COMPARISONS WITH THE EXAMPLE PROFILES DRAWN SOLID.
Example :- A: Ref. Profile (perfect form and position) B: Spacing error C: Profile error (deviation from true involute) D: Lead error E: Radial position error (caused by runout) F: Tooth thickness error

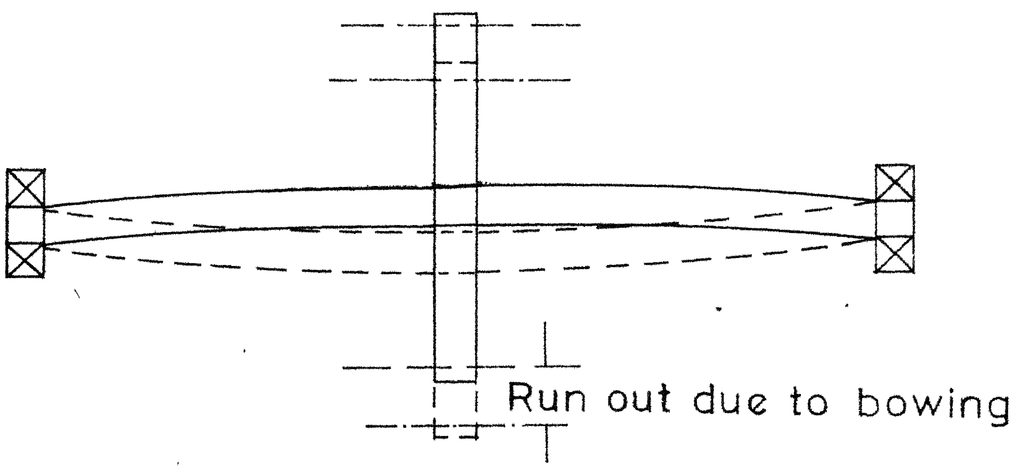


FIG. B.2 SHAFT RUNOUT AT GEAR MOUNTING.

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